AN EMPIRICAL ANALYSIS OF HEDGE RATIO:

THE CASE OF NIKKEI 225 OPTIONS

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ABSTRACT

The seminal paper by Black and Scholes (1973) has formulated the closed-form optimal hedge ratio of option contract, delta, based on many restrictive assumptions. The Black and Scholes model belongs to the family of parametric models. This thesis aims at estimating such optimal delta by employing various econometric methods, namely, the local polynomial fitting and local parametric fitting. These estimation techniques belong to the family of nonparametric models and are compared with our benchmark parametric model, the Black and Scholes model. We use Nikkei 225 index and its options contracts (traded in Osaka Security Exchange) as an illustration. The performance of the models is evaluated in terms of "tracking error" across different levels of maturity and ratio between spot price and spot price (moneyness).

The in-sample evaluation shows that local polynomial models are far inferior to local parametric models in general. The superiority of local parametric models, however, decreases with both moneyness and maturity. This suggests the importance of the Black and Scholes model used in nonparametric estimation especially for small maturity and moneyness options.

In additional, best out-of-sample tracking records are obtained by combining deltas from nonparametric models and the Black and Scholes model. The optimality of this combination indicates that our nonparametric approach to options analysis and traditional derivatives analysis exploit complementary information.

摘要

Black and Scholes (1973)在其具啓發性的論文中，以很多限制性的假設爲基礎，確切地計算出期權合約的最佳對沖比率(delta)。Black and Scholes 的模型屬於參數模型。然而，本論文應用計量經濟學中的非參數模型，包括區間多項式擬合法及區間係數擬合法，來估計期權合約的最佳對沖比率，並與我們的標準參數模型(Black and Scholes 模型)作比較。本研究以日經平均指數及其在大板証卷交易所的期權合約作其研究對象。文中模型的表現均以「軌跡誤差」(tracking error)，在不同的限期(maturity)及現貨價與行使價的比例(moneyness)分類評估。

一般來說，樣本內的評估顯示區間多項模型的表現遠遠較區間係數模型遜色。區間模型的優勢則隨限期及現貨價與行使價的比例上升而減弱。這可反映到 Black and Scholes 模型在非參數估計中，特別是對於短限期及低現貨價與行使價比例的期權的重要性。

另外，最佳樣本外的軌跡誤差紀錄是透過結合非參數模型及 Black and Scholes 模型的最佳對沖比例所得。這優勝的方法顯示非參數模型在期權分析及傳統的衍生工具分析均有其互補的資料。

CONTENTS

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CHAPTER ONE

INTRODUCTION

Derivative analysis studies the pricing and hedging of financial instruments whose value depends on those of a limited set of underlying securities. Prime examples of such instruments are *options*. An option is a security, which gives its holder the *right* to trade in a fixed number of specified underlying assets at a predetermined *exercise price*, either only at a fixed maturity (*European options*) or anytime before maturity (*American options*). A *call* option gives the holder the right to buy the underlying securities while a *put* option gives the holder the right to sell them. Traders who have bought options enter a *long position* whereas those who have sold or written options enter *a short position*. The *call price* and the *put price* refer to the cost to acquire a call option and a put option, respectively.

Fundamentally, all options can be categorized as either *financial options*, where the underlying assets are financial assets such as stocks, foreign currencies or futures contracts, or *non-financial options*, where the underlying assets are non-financial assets. So, stock index option is a financial option where the underlying asset of this type of option is stock index (for example, Standard and Poor's 100 index, Standard and Poor's 500 index and Nikkei 225 stock average index). Actually, index options are settled in cash rather than by delivering the securities underlying the index. This means that, upon exercise of an option, the holder of a call option receives the intrinsic value of a call option, which is equal to the value of the index at the time of exercise minus the exercise price in cash and the writer of the option pays this amount in cash. Similarly, the holder of a put option receives the intrinsic value of a put option, which is equal to the exercise price minus the value of the index at the time of exercise and the writer of the option pays this amount in cash. Each contract is for certain times the value of the index.

From humble beginnings in little notices over-the-counter markets, standardized listed stock options are first traded on the Chicago Board Options Exchange (CBOE) in 1973. Since then option markets grow dramatically. The first index option, Standard and Poor's 100 Index Option, is introduced by the CBOE on March 11, 1983. In the same year, the CBOE introduces options on the Standard and Poor's 500 Index. Since then, stock index options have gained much popularity. The Osaka Security Exchange (OSE) introduces American Style call and put options on the price-weighted Nikkei 225 Index on June 12, 1989.

The main reasons for the success of stock index options are that they are capable of managing large stock portfolios and controlling price uncertainty through hedging. Compared with forward or future contract, option contract is more flexible because it gives the holder only the right but not the obligation to trade. Moreover, the option contract size is typically smaller. These two facts contribute to the popularity of stock index options.

Stock index options serve as an alternative mean of investment for investor in mimicking the underlying stock index. For instance, in order to minimize the risk of a long (short) position in the spot market, an investor would generally enter a short (long) position in the option market. By adopting this hedging strategy the traders' loss in the spot market can be compensated by the gain in the option market and vice versa. However, we have to think of a critical factor, the *hedge ratio* – how many underlying assets that a trader should enter for a particular position in the option market so as to form a risk-free portfolio.

Intuitively, hedge ratio should be equal to one, which means that a short (long) position of Q units in the option market on an asset can be hedged by a long (short) position of Q units in the spot market of that asset. The hedging scheme is an example of a *static hedging scheme* or sometimes also referred to as a *hedge-and-forget hedging scheme* where the hedge, once set up, is never adjusted. Apparently, this hedging scheme is not applicable in reality because static hedging using options is to provide one-side tracking, and is only good for the position of the maturity (end of the hedging horizon) not any time prior to maturity.

Black and Scholes (1973) first introduces the hedge ratio, *delta*, of an option as the partial derivative of the call price with respect to the price of underlying asset, that is the rate of change of the option price with respect to the price of underlying asset. They show that it is possible to set up a riskless portfolio, consisting of a position in the underlying asset and a position in the option contract, which is *delta-neutral.* Since the delta of the underlying asset is 1.0, one way of doing so is to take a position in the underlying asset which equals the minus delta of the portfolio being hedged. The partial derivative is then of great interest because it determines the delta. Since the delta of a portfolio changes over time, the position in the underlying asset should also be frequently adjusted to maintain a delta-neutral position. This implies that a *dynamic hedging scheme* should be used instead. It is then natural to ask the question of what the *optimal delta* is and how it is determined econometrically.

The seminal thesis by Black and Scholes (1973) has formulated the closed-form option hedge ratio, delta, based on many restrictive and yet unrealistic assumptions.

Focusing on the continuous-time modeling, one can immediately mention the impossibility of hedging in continuous time. Discretization-induced tracking errors and hence model misspecification are induced from blindly applying continuous-time modeling to a discrete-time setting. Clearly, the traditional Black and Scholes model to pricing and hedging derivative securities belongs to a family of parametric models, which relies on particular functional forms of the underlying variables.

On the other hand, various nonparametric methods, which relax these restrictive parametric assumptions such as lognormality or sample-path continuity, are proposed in the literature to determine the optimal delta. Nonparametric models are widely accepted because we do not need to specify the relationship among variables and are robust to the specification of functional forms.

Local polynomial model is one of the nonparametric models, which approximate the relationship among underling variables locally with a polynomial. Fan (1993) and Hastie and Loader (1993) discover that local polynomial fitting possesses nice theoretical results such as minimax optimality in efficiency, automatic boundary correction, design adaptation, and simple bias and variance expressions. However, the shortcoming of such design-adaptive model is that it is highly data-intensive and will be an obstacle for the application of other nonparametric estimation methods.

Another kind of nonparametric models is *local parametric model*: one fits parametric models locally, in that way exploiting at a maximum insight of standard derivatives theory as far as curvature (convexity) is concerned. Such approach combines the merits of both parametric and nonparametric models and has recently been suggested by Hjort (1995).

The objective of this thesis is to apply both local polynomial and local parametric models to estimate the optimal delta, using Nikkei 225 index and its options (traded in Osaka Security Exchange, OSE) as an illustration. The results are evaluated by comparing their hedging performance in terms of "tracking error" with that of the parametric delta.

The remainder of this thesis is organized as follows. Chapter 2 provides a review of the literature. Chapter 3 presents the parametric and nonparametric models that will be applied in this thesis. Chapter 4 gives a brief description of data. The estimation and evaluation results are presented in chapter 5. Chapter 6 concludes the thesis.

CHAPTER TWO

REVIEW OF THE LITERATURE

This chapter is divided into two parts. The literature on the parametric models (the Black and Scholes model) will be reviewed in the first part while the literature on the nonparametric models will be discussed in the second part. Detailed frameworks of the models and the estimation procedures will be presented in next chapter.

Parametric Models

Parametric models specify the functional forms of and hence the numerical relationships among the underlying variables explicitly. In the context of our analysis, we use the Black and Scholes model as our benchmark parametric model for comparison. However, if the postulated functional forms are different from the actual relationships, such rigidity could lead to poor and misleading estimation results.

Black and Scholes Model

Black and Scholes (1973) develops the classical option pricing formula, by which exact solutions for European call and put options on a non-dividend-paying stock can be obtained. To derive the formula for the value of an option in terms of the price of the stock, they have asserted many "ideal" conditions in the stock market and option market. Two important assumptions about the stock price dynamics are made: first: the stock price follows a continuous path through time and second, the instantaneous volatility and riskless rate is nonstochastic. Besides, they assert that the stock price follows a geometric Brownian motion. This is analogous to assuming that the stock price follows a random walk in continuous time with a variance proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite interval is lognormal. The variance of rate of the return on the stock is constant. Consequently, they formulate the closed-form option pricing formula by the risk-neutral valuation.

In addition, they also introduce the hedge ratio, *delta*, of an option for a riskless portfolio consisting of a position in the underlying asset and a position in the option contract as the partial derivative of the call price with respect to the price of underlying asset.

Since the seminal paper of Black and Scholes, various models are proposed in the literature where one or both of the two restrictive Black and Scholes assumptions are relaxed. Examples include (i) the stochastic-interest-rate option models of Merton (1973) and Amin and Jarrow (1992); (ii) the jump-diffusion/pure jump models of Bates (1991), Madan and Chang (1996), and Metron (1976); (iii) the constant-elasticity-of-variance model of Cox and Ross (1976); (iv) the stochastic-volatility models of Heston (1993), Hull and White (1987), Melino and Turnbull (1990, 1995), Scott (1987), Stein and Stein (1991), and Wiggins (1987); (v) the stochastic-volatility and stochastic-interest-rates models of Amin and Ng (1993), Bailey and Stulz (1989), Bakshi and Chen (1997a,b), and Scott (1997); and (vi) the stochastic-volatility jump-diffusion models of Bates (1996a,b) and Scott (1997). (vii) the GARCH option pricing model of Duan (1995). It is a discrete-time model and allows for stochastic volatility. It also addresses both of the two restrictive Black and Scholes assumptions. All of these parametric models rely heavily on particular functional forms of the underlying variables.

However, in standard Black and Scholes model, the parameters of the continuous-time value processes are difficult to estimate. Even if the relationship between the available time-series and underlying processes are realized, actual estimation has revealed plenty of problems. Gibbons and Ramaswamy (1993) discovers the problem of near-unit-root behavior of continuous-time interest rate while Bossaerts and Hillion (1993) discovers the problem of near-unit-root behavior of stochastic volatility. Lo and Wang (1995) also introduces the problem of lack of precision in the estimation of the mean return, which is relevant when the data come in discrete time.

Therefore, apart from examining the stochastic nature of riskless rate of interest and the volatility of the underlying asset, this thesis examines the errors from blindly applying continuous-time modeling to a discrete-time setting. The idea is to formulate the derivatives problem by a hedge equation, which is estimated directly with flexible and yet robust procedures. The estimation techniques are known as local polynomial estimation and local parametric estimation: one approximates the function locally with a polynomial and a parametric model respectively, thereby exploiting at a maximum insight of standard derivatives theory as far as curvature (convexity) is concerned.

To gauge the practical relevance of local polynomial estimation, we will follow Hutchinson, Lo and Poggio (1994) and Bossaerts and Hillion (1997) to evaluate the performance of nonparametrically estimated hedges and that of Black and Scholes hedges by using Nikkei 225 index and its options as an illustration. In the application of the Black and Scholes model, the following two findings are also noteworthy: First, Bailey (1989) applies the Ramaswamy and Sundaresan (1985) model to the valuation of Nikkei 225 and Osaka Stock 50 future contracts. The results indicate that the volatility of Japanese interest rates is so low that there is no incremental impact on futures pricing. Therefore, we can simply assume that the riskless interest rate, Gensaki (repo) rate or Euroyen deposits rate as in the case of Japan, satisfies the nonstochastic feature. Second, Chan and Karolyi (1991) shows that Nikkei returns are well fitted by a GARCH model with a time varying volatility.

Despite the fact that the Black and Scholes model is generally not used in its original form in practice, we focus on it here because it is still a widely used benchmark model and serves as an example of a parametric model whose assumptions are questionable in the context of our thesis.

Nonparametric Estimation Techniques

Unlike the parametric estimation, nonparametric estimation technique can be used to capture a wide variety of nonlinearities without recoursing to any one particular specification of the nonlinear relationship. Therefore, this technique is widely discussed in the literature. One of the reasons for the success of financial application of nonparametric technique is that it requires fewer assumptions about the nature of nonlinearities, which is contrasting to the highly structured parametric approach.

However, nonparametric estimation is highly data-intensive and requires larger sample size. Moreover, overfitting is another serious problem that affects the effectiveness of nonparametric estimation. This occurs when a model fits "too well" in the sense that the model has captured both random noise and genuine nonlinearities as well.

To have a better understanding of this technique, suppose the observations $\{(X_1, Y_1),...,(X_n, Y_n)\}$ are generated by the model

$$Y_i = m(X_i) + \sigma(X_i)\varepsilon_i, \qquad \text{for } i = 1, 2, \dots n,$$

where $E(\varepsilon_i) = 0$ and $Var(\varepsilon_i) = 1$, and X_i and ε_i are independent. Note that $m(x_i) = E(Y_i | X_i = x_i)$ and $\sigma^2(x_i) = Var(Y_i | X_i = x_i)$, which allows for possible heteroscedasticity. Compared with parametric models, nonparametric models always have much less restrictive assumptions about the functional forms and the distributions of the interested quantities. Therefore it can avoid many problems encountered in parametric models.

Basically, the nonparametric estimators are smoothing estimators. They are done by minimizing the observational errors using averaging in sophisticated ways. Nadaraya (1964) and Watson (1964) first propose to estimate a function by local weighted averaging. Furthermore, kernel regressions, orthogonal series expansion, projection pursuit, nearest-neighbour estimators, average derivatives estimators, splines, and artificial neutral networks are all examples of smoothing. Here, we discuss the literature of the nonparametric estimation techniques that are relevant to the thesis.

Kernel Estimation

The original studies of Nadaraya (1964) and Watson (1964) propose to estimate the conditional mean function $m(x)$ by local weighted averaging. The corresponding Nadaraya-Watson (NW) kernel regression estimator is given by

$$\hat{m}_h^{NW}(x) = \frac{\sum_{i=1}^{n} K_h(X_i - x)Y_i}{\sum_{i=1}^{n} K_h(X_i - x)}$$

where K is a symmetric real-valued function assigning weight and is called a *kernel function*. The parameter h is called a *bandwidth* or a *smoothing parameter* which is a nonnegative number controlling the size of the local neighborhood. $K_h(\cdot) = K(\cdot/h)/h$ is a rescaling function of K.

Gasser and Müller (1979) proposes a similar estimator. They suggest that the random denominator of NW estimator is inconvenient when taking derivatives of the estimator and deriving its asymptotic properties. Hence, by assuming that the data have already sorted according to the *X*-variables. The proposed Gasser-Müller (GM) kernel regression estimator is then given by

$$\hat{m}_h^{GM}(x) = \sum_{i=1}^{n} \int_{s_{i-1}}^{s_i} K_h(u - x)du Y_i,$$

where $s_0 = 0$, $s_i = (X_i + X_{i+1})/2$, $i = 1, \ldots, n - 1$ and $s_n = 1$.

From a function approximation point of view, both Nadaraya-Watson and Gasser-Müller estimators use local constant approximation. Unfortunately, both of them suffer from serious drawbacks. Indeed, the NW estimator suffers from large bias particularly in the region where the derivative of the regression function or of the design density (i.e. the density of the random variable *X*) is large, even when the true regression curve is linear. The Gasser-Müller estimator on the other hand corrects the bias of the NW estimator but at the expense of increasing its variablility. Further, both estimators have a large order of bias when estimating a curve at the boundary region.

Local Polynomial Estimation

One way to repair the drawbacks of the Nadaraya-Watson and Gasser-Müller estimators is to use a higher-order approximation, that is, to approximate the regression function by a polynomial. The idea of local polynomial estimation has been around for a long time and is actually one of the nonparametric regression approaches to deal with the nonlinearity of the regression function. For the local polynomial fitting, we fit low-order polynomial in x locally at x_0, and the estimate of $m(x_0)$ is taken from the fitted polynomial at x_0. The size of the local neighborhood, the bandwidth, can be chosen either subjectively by analysts or objectively by data.

The classical works of Stone (1977) and Cleveland (1979) provide the building blocks for the development of such techniques. Stone (1977) systematically studies the asymptotic properties of nonparametric regression. Cleveland (1979), on the other hand, applies polynomial fitting locally and develops a procedure known as LOWESS (LOcally WEighted Scatter plot Smoothing) that avoids distortions resulting from outliers. Two later works of Stone (1980, 1982) study the rates of convergence for local regression in details. Recent works on local polynomial fitting include Fan (1993), Fan and Gijbels (1992) and Ruppert and Wand (1994).

The local polynomial, as noted in Hastie and Loader (1993), and Fan (1993), has many advantages over the other nonparametric regressions. For example, it can adapt to various types of designs such as random and fixed designs, highly clustered and nearly uniform designs. It corrects the boundary bias automatically without increasing the variance of the estimator. These features are particularly important in practical issues.

In addition, Fan (1993) shows that local polynomial, particularly linear, fitting attains high asymptotic minimax efficiency properties among other linear estimators including the previously discussed Nadaraya-Watson and Gasser-Müller estimators. This minimax efficiency is measured in terms of linear minimax risk. To put it simple, a 67% efficient estimator uses only about 67% of the available observations in estimation. That means, such an estimator based on a sample of size 100 performs equivalently to the best linear estimator (which has 100% efficiency) with sample size 67. Table 1 gives a comparison of such minimax efficiency among several linear estimators with different kernel functions.

Generally, local polynomial fitting has certain advantages over the NW and GM estimators not only for the regression curves estimation, but also for the derivative estimation. We are interested in the derivative estimation because the first and second derivatives of a regression function often have important implications. Gasser and Müller (1984) modifies the result of Gasser and Müller (1979) to estimate the derivatives of the function. However, with the aid of Taylor's expansion, local polynomial fitting provides a much intuitive and convenient way for derivative estimation. Müller (1987) establishes an asymptotic equivalence between higher order kernel functions and local polynomial fitting with a well-behaved design.

Fan, Gijbels, Hu and Huang (1996) shows that in order to estimate the v^{th} derivative of a function, it is optimal to fit a local polynomial of degree p such that $p-v$ is odd. Besides, for a given bandwidth, a large value of p would expectedly reduce the modeling bias, but would cause a large variance and a considerable computational cost. Since the modeling bias is primarily controlled by the bandwidth, Fan and Gijbels (1996) recommends to use of the lowest odd order, i.e. $p=v+1$, or occasionally $p=v+3$. Similar results are obtained in Ruppert and Wand (1994) and the discussion is extended to a multivariate setup in their work.

Liang (1994) approximates the optimal hedge ratio in derivative hedging by means of polynomials and estimates the parameters by least squares. For his procedure to recover the correct parameters, however, the degree of polynomial has to increase with the sample size.

Bossaerts and Hillion (1997) applies local polynomial fitting to option analysis. By fitting linear and quadratic functions locally, they find that the performance of the local polynomial estimates is inferior to that of a local parametric model. Consequently, they suggest that a local parametric estimation technique, which is done by fitting a parametric model (the Black and Scholes model) locally, should be used instead to evaluate the hedging performance in discrete time.

Local Parametric Estimation

Now we discuss another nonparametric estimation, namely local parametric estimation. This approach is an extension of the previous local polynomial estimation. Local polynomial modeling approximates the regression function locally by a polynomial function, while local parametric modeling approximates the regression function locally by a parametric model.

With the knowledge of local parametric estimation, it is intuitive to think that this approach should be particularly appealing when one utilizes nonparametric technique to analyze a data set with much prior knowledge on its functional form. Therefore, local parametric estimation is particularly of interest under the context of derivative analysis because one can utilize the proposed parametric model at a maximum insight of standard derivatives theory as far as nonlinearity is concerned.

Hjort (1995) applies local parametric estimation in the context of duration model. In addition, Bossaerts and Hillion (1997) applies local parametric estimation to option analysis. They compare the performance of Black and Scholes hedge ratios against those obtained from local parametric estimation. In the latter, the weights of the duplicating portfolio are estimated by fitting parametric model (Black and Scholes model) in the neighborhood of the moneyness and maturity of derivatives. They show that the errors from Black and Scholes hedging in discrete time are serious for those options whose return is nonlinearly related to that of the underlying security, especially for short-maturity and out-of-money options.

In this thesis, we will follow closely the local parametric technique in option analysis established by Bossaerts and Hillion (1997) under both arbitrage and no-arbitrage conditions. However, it should be noted that the hedge elasticity, or usually referred to as the hedge portfolio weight, derived from the hedge equation should then be converted to hedge ratio for the evaluation of the hedging performance

Many other nonparametric estimation methods for pricing and hedging derivative securities have been proposed in the literature, such as radial basis function networks, multilayer perceptron networks and projection pursuit. Empirically, these learning networks can outperform the Black and Scholes model. Yet, they are not considered in this thesis and thus are not discussed in details here.

Bandwidth Selection

Before we go into the details of our analytical framework, we have to discuss two issues which are crucial to all nonparametric estimations, namely *bandwidth selection* in this section and *kernel function* in the following section.

As mentioned, nonparametric estimation techniques involve locally weighted regressions. The problem is how large the local neighborhood should be and the size of local neighborhood is controlled by a parameter, the bandwidth. When the bandwidth is small, few observations within the neighborhood will be used for the estimation at each particular point. The estimated function will be very "bumpy" and similar to the interpolation of the observations. On the other hand, when the bandwidth is larger, more observations are considered and the resulting estimator will be "smoother". Indeed, if the bandwidth is so large to covers the whole range of observations, nonparametric estimation makes no difference from the parametric one. Therefore, the performance of the estimator largely depends on the bandwidth selected.

Precisely, the selection of bandwidth involves the tradeoff between bias and variance of the estimator – the larger the bandwidth, the larger the bias and smaller the variance. Certainly, the bandwidth can be chosen subjectively. However, it is better to use some objective criteria. In particular, a data driven (automatic) bandwidth can be obtained by minimizing the mean squared error (MSE) or the mean integrated squared error (MISE) of the estimator. However, these criteria usually involve unknown quantities which need to be further estimated. This then leads to various methodologies of derivation of the bandwidth.

Silverman (1986) suggests a simple bandwidth selection method for the kernel density estimator, the rule of thumb (ROT) method which is with reference to the normal distribution and assumes that the distribution of the observations is normal. The idea can be dated back to Deheuvels (1977) who proposes it for his histograms and Scott (1979) who discusses the choice of the optimal bin width for histogram. Fan and Gijbels (1995b) extends to the application in the local polynomial fitting. Härdle and Marron (1995) develops another ROT bandwidth where a piecewise polynomial fitting is used to recover the curvature. Bowman (1984) finds that a ROT bandwidth performs very well in his Monte Carlo study when the underlying density is close to normal.

Rudemo (1982) and Bowman (1984) propose the least square cross-validation bandwidth. It involves minimizing the least square cross-validation function. For a given datum point i, we use data $\{(X_j, Y_j), j \neq i\}$ to build the regression function. The criterion is defined by the weighted average of squared errors of the regression function. Härdle (1990) carries the idea over to the estimation of derivative curves. Also, other versions of cross-validation method are proposed in which different cross-validation functions are minimized. For examples, Scott and Terrell (1987) proposes the bias cross-validation bandwidth. Wahba (1977) and Craven and Wahba (1979) use the generalized cross-validation. Sheather (1992) and Fan and Gijbels (1996) show that the performances of the bandwidths under different cross-validation methods are similar.

The direct plug-in method introduced by Woodroofe (1970) in density estimation is another popular bandwidth selector. The idea involves substitution of the unknown quantities by pilot estimators and then carrying out an iterative procedure is carried out until convergence is attained. Scott, Tapia and Thompson (1977) proposes an iterative procedure for the estimation of the plug-in estimated bandwidth and Rupper, Sheather and Wand (1995) applies plug-in techniques in the regression estimation setup with realistic examples. Sheather and Jones (1991) develops another type of plug-in method by using the bandwidth in initial setup and making reference to a normal density with unknown standard deviation.

Gasser, Kneip and Köhler (1991) presents a plug-in estimator which builds on estimation of the asymptotically optimal bandwidth from the data and is iterative in nature. They do so by directly plug-in estimators for the residual variance and for an asymptotic expression for the bias into the asymptotic formula. The functional that quantifies bias is approximated by the integrated squared second derivative of the regression function. This plug-in estimator of the bandwidth has much lower variability than cross-validation estimators for various situations including nonsmooth functions. It can also be extended to estimate the optimal bandwidth when determining derivatives of a regression function, probability densities and spectrum densities

Many other methods are proposed as alternate bandwidth selectors. As a result, we inevitably have to be selective. Moreover, not all of them are popular and have good performance. Sheather (1992) compares performance of six popular bandwidth selection methods. Fan and Gijbels (1995a) studies the data-driven bandwidth selection in local polynomial fitting.

In this thesis, least square cross-validation (CV) method and plug-in (PI) method proposed by Gasser, Kneip and Köhler (1991) will be used to choose the bandwidth.

Kernel Functions

After having studied the methods of local estimation and the choice of bandwidth, we now come to determine the weighting scheme, the kernel function K, to each of the data. In general, it is a symmetric probability density function

Marron and Nolan (1988) states that Guassian kernel and those kernels derived from the symmetric Beta family are the most widely used kernel functions. The Gaussian kernel is defined as:

$$K(u) = \left(\sqrt{2\pi}\right)^{-1} \exp\left(-u^2/2\right),$$

while the symmetric Beta family is defined as:

$$K(u) = \frac{1}{Beta(1/2, \gamma+1)}\left(1-u^2\right)^{\gamma} I(|u| \le 1),$$

where $I(\cdot)$ is the indicator function. The function $Beta(\cdot,\cdot)$ generates a constant so that K is a density function. When γ is equal to 0, 1, 2 and 3, the function above will generate the uniform, Epanechnikov, biweight (quartic) and triweight kernel functions respectively. The constant attached to the function $\left(1-u^2\right)^{\gamma} I(|u| \le 1)$ will be 1/2, 3/4, 15/16 and 35/32 respectively. In fact, this family includes the Gaussian kernel function in the limit as $\gamma \to +\infty$.

Fan, Gasser, Gijbels, Brockmann and Engel (1995) has proved that Epanechnikov kernel function is optimal in the sense of minimizing the variance of the estimators. However, as noted in Fan and Gijbels (1996), the choice of the kernel K is not essential to the performance of the resulting estimators, both theoretically and empirically. Table 2 shows the relative performance of different kernel functions to

Epanechnikov kernel function in terms of asymptotic MISE.

Although the Epanechnikov kernel function is always considered as a benchmark of efficiency, biweight kernel is widely used in most statistical softwares specialized for nonparametric estimation due to its simplicity. In this thesis, both Epanechnikov and biweight kernel functions will be used to assign weights to observations in estimation.

CHAPTER THREE

METHODOLOGY

This chapter consists of two parts. In the first part, we discuss the parametric model, the Black and Scholes model. The second part introduces the frameworks of the nonparametric models and the related issues. All models presented here will be applied to estimate the optimal delta.

Parametric Models

Black and Scholes model

Given the power and flexibility of nonparametric estimations to approximate complex nonlinear relations, a natural application is to derivative securities whose pricing formulas are highly nonlinear. While the accuracy of the nonparametric models is obviously of great interest, this alone is not sufficient to ensure the practical relevance of our nonparametric approach. In particular, the ability to hedge an option position is important since the existence of an arbitrage-based pricing formula is predicted upon the ability to replicate the option through a dynamic hedging strategy. In this thesis, we will investigate the delta-hedging errors explicitly in our empirical application and use Black and Scholes model as our benchmark parametric model for comparison.

Black and Scholes (1973) formulates the classical option pricing formula by which exact solutions for European call and put options on a non-dividend-paying stock are obtained. The derivation details of this model are presented in Appendix. By risk-neutral valuation, the closed-form option pricing formula of European call option, denoted as $C(t)$, is:

$$C(t) = S(t)N(d_1) - Xe^{-r\tau}N(d_2); \tag{1}$$

where

$$d_1 = \frac{\ln(S(t)/X) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \quad d_2 = d_1 - \sigma\sqrt{\tau},$$

$N(\cdot)$ is the standard normal cumulative distribution function and the parameters, $S(t), X, \tau$ and σ are the underlying asset price, the exercise price, the time-to-maturity, the riskless rate of interest and the volatility of the underlying asset (i.e. the standard deviation rate of the underlying asset) respectively.

As mentioned before, the hedge ratio, *delta*, of an option is defined as the partial derivative of the call price with respect to the underlying asset price. Black and Scholes show that one can set up a riskless portfolio consisting of a position in the underlying asset which equals the minus delta of the portfolio being hedged and a position in the option contract. By doing so, the position of such portfolio will become *delta-neutral* because the delta of the underlying asset is 1.0. Apparently, the delta is crucial to the success of option hedging and is indeed ever changing over time. Thus, by taking the partial derivative of equation (1), the delta of a European non-dividend-paying call option, denote as Δ, is

MODEL BS $$\Delta = N(d_1). \tag{2}$$

Kernel Estimation

Nadaraya-Watson (NW) estimator

As mentioned in Chapter two, the Nadaraya-Watson (NW) kernel estimator for the mean response $m(x)$ is given by:

$$\hat{m}_h^{NW}(x) = \frac{\sum_{i=1}^{n} K_h(X_i - x)Y_i}{\sum_{i=1}^{n} K_h(X_i - x)}; \tag{3}$$

where $K_h(\cdot) = K(\cdot/h)/h$, and h is the smoothing parameter (i.e. the bandwidth) which controls how "smooth" the regression function is. The NW estimator indeed comes from the *locally* weighted least square problem:

$$\hat{m}(x) = \hat{\theta} = \arg\min_{\theta} \sum_{i=1}^{n} (Y_i - \theta)^2 K\left(\frac{X_i - x}{h}\right).$$

Gasser-Müller (GM) estimator

Recall from Chapter two, the Gasser-Müller (GM) kernel estimator for the mean response $m(x)$ is given by:

$$\hat{m}_h^{GM}(x) = \sum_{i=1}^{n} \int_{s_{i-1}}^{s_i} K_h(u - x) du Y_i; \tag{4}$$

where $s_0 = 0$, $s_i = (X_i + X_{i+1})/2$, $i = 1, \ldots, n - 1$ and $s_n = 1$.

Local Polynomial Estimation

Local polynomial estimation is an extension of the NW technique. The key difference is that local polynomial estimation approximates the unknown regression function $m(x)$ locally by a polynomial of order p instead of using a local constant approximation. The canonical representation of the derivative securities problem used by Bossaerts and Hillion (1997) is:

$$\left(\frac{\Delta C}{C}\right)_j = a_1(x) + b_1(x)\left(\frac{\Delta S}{S}\right)_j + \varepsilon_j ; \qquad (5)$$

where C denotes the call price and ΔC denotes the change in call price. S denotes the value of the underlying asset. We refer the underlying asset in this thesis to the 'stock index'. ΔS denotes the change in the stock index.

The power of the above analysis lies in the allowance of the hedge portfolio weights (i.e., the regression coefficients) to vary over time and they are functions of the available information, such as moneyness, maturity, interest rate, etc. Accordingly, we have been writing the coefficients $a_1(x)$ and $b_1(x)$ as a function of x to indicate that they are functions of yet unspecified information.

The slope coefficient $b_1(x)$ determines the *hedge portfolio weight* of the underlying asset in the replicating portfolio, with the remainder $1-b_1(x)$ invested in a one-period riskless asset at the rate of return equals to γ.

We have written equation (5) in terms of *returns*. Usually, however, theoretical derivatives pricing problems are formulated in terms of *payoffs*. Once we have realized that we are going to estimate the hedge portfolio weight, the former presentation is preferable because the values of the underlying assets are likely to be nonstationary. For instance, they could follow geometric Brownian motions. In that case, returns are stationary, but the payoffs on the underlying assets and those of a derivative with fixed moneyness will be nonstationary. This induces the difficulty of the theoretical-statistical analysis of estimation of equation (5) if they are written in terms of payoffs.

Within the context of optimal hedge portfolio weight estimation established by Bossaerts and Hillion (1997), if local linear estimation is used, the minimization problem at x (some location with the range of $(\Delta S/S)_j$'s) is defined as:

MODEL N1 $$\underset{a_1(x),b_1(x)}{Min} \sum_{j=1}^{n}\left\{(\Delta C/C)_j - a_1(x) - b_1(x)(\Delta S/S)_j\right\}^2 K\left(\frac{(\Delta S/S)_j - x}{h}\right); \quad (6)$$

where $a_1(x)$ and $b_1(x)$ are the coefficients optimal at x. In particular, $b_1(x)$ is the *local* optimal hedge portfolio weight under this approach and should then be converted to hedge ratio in order to facilitate the evaluation of hedging performance.

Theoretically, x can be *any* point within the range of the realization of the independent variable $(\Delta S/S)_j$. In practice, the range of the regressor is divided into a certain number of grid points and evaluation of equation (6) is made at each of them.

With Taylor's expansion, a regression function $m(x)$ can be approximated locally by:

$$m(x) \approx \sum_{j=0}^{p} \frac{m^{(j)}(x)}{j!}(z-x)^j \equiv \sum_{j=0}^{p} \beta_j (z-x)^j,$$

for z in a neighborhood of x. This model $m(x)$ is fitted locally by a simple polynomial model and this suggests a locally weighted polynomial regression to estimate the derivatives:

$$\underset{\beta_j(x)}{Min} \sum_{i=1}^{n}\left\{Y_i - \sum_{j=0}^{p} \beta_j(x)(X_i - x)^j\right\}^2 K\left(\frac{X_i - x}{h}\right),$$

where $\hat{\beta}_j(x)$ are the minimizers of the equation above and depend on the location of x. The estimator of the v^{th} derivative of $m(x)$ is then given by:

$$\hat{m}^{(v)}(x) = v!\,\hat{\beta}_v .$$

Fan and Gijbels (1996) shows that the local v^{th} derivative can be best estimated with local polynomial of degree v+1, v+3 and so on. From the regression relationship between $(\Delta C/C)_j$ and $(\Delta S/S)_j$ (derived by Bossaerts and Hillion (1997)), the hedge portfolio weight can be thought as the first derivative: $\frac{\partial(\Delta C/C)_j}{\partial(\Delta S/S)_j}$. Therefore, we can also estimate the local optimal hedge portfolio weight with polynomial of degree 2, 4, and so on. In this thesis, polynomial of degree 2 is also considered. The minimization problem then becomes:

MODEL N2

$$\underset{a_2(x),b_2(x),c_2(x)}{Min} \sum_{j=1}^{n} \left\{(\Delta C/C)_j - a_2(x) - b_2(x)(\Delta S/S)_j - c_2(x)(\Delta S/S)_j^{\ 2}\right\}^2 K\left(\frac{(\Delta S/S)_j - x}{h}\right), \tag{7}$$

and the local optimal hedge ratio at x is then found by converting the estimator of $b_2(x)$.

Both local linear estimation (model N1) and local quadratic estimation (model N2) will be applied 1in this thesis to obtain the optimal delta.

Local Parametric Estimation

Similar to local polynomial estimation, local parametric estimation is done by approximating the regression function locally by a parametric model. In particular parametric cases, the replicating error can be reduced to zero by a judicious choice of $a_1(x)$ and $b_1(x)$. Within the context of option analysis, the prime example is Black and Scholes (1973) model where the derivative is a call option written on common stock.

Denote m as the option's moneyness, i.e. its stock price divided by the exercise price, and τ as its maturity. Let r be the interest rate which is a positive constant, and σ be the instantaneous volatility of the stock price. Set[1]

$$b_1(m,\tau;\sigma)=\frac{1}{1-\frac{e^{-r\tau}}{m}\frac{N(d_2(m,\tau;\sigma))}{N(d_1(m,\tau;\sigma))}}, \tag{8}$$

and $a_1(m,\tau)=(1-b_1(m,\tau))\gamma$, for some $\gamma > 0$. $N(\cdot)$ denotes the standard normal distribution function, and

$$d_1(m,\tau;\sigma)=\frac{\ln(m)+(r+\sigma^2/2)\tau}{\sigma\sqrt{\tau}},\quad d_2(m,\tau;\sigma)=d_1-\sigma\sqrt{\tau}.$$

This choice for the hedge portfolio weight eliminates the tracking error provided: (i) the stock price follows a geometric Brownian motion, and (ii) the hedging interval is infinitesimal. If there are no arbitrage opportunities, $\gamma=r$.

Now consider fitting Black and Scholes hedge portfolio weight (equation (8)) locally to obtain $b_1(m,\tau)$. To estimate $a_1(m,\tau)$, we fit one minus the Black and Scholes weight function times a constant, γ. We thereby minimize the weighted sum of squared error for a data set of n observations $((\Delta C/C)_j,(\Delta S/S)_j,m_j,\tau_j)$ for j = 1,…,n in the following equation with respect to two parameters, σ and γ:

$$\left(\frac{\Delta C}{C}\right)_j=(1-b_1(m_j,\tau_j;\sigma))\gamma+b_1(m_j,\tau_j;\sigma)\left(\frac{\Delta S}{S}\right)_j+\varepsilon_j; \tag{9}$$

where m_j and τ_j denote the option's moneyness and maturity respectively, for observation j.

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[1] $b_1(m,\tau;\sigma)$ is usually referred to as the hedge elasticity, which is the hedge ratio divided by the option premium. By Itô's lemma, the hedge ratio is the derivative of the theoretical call price with respect to the price of the underlying asset. Equation (8) is then obtained after substituting the analytical call price formula for the call price in the definition of the hedge elasticity.

If $\hat{\gamma}$ and $\hat{\sigma}$ denotes the optimal estimates of γ and σ respectively, the estimates of $a_1(m,\tau)$ and $b_1(m,\tau)$ are defined as $\left(1-b_1\left(m,\tau;\hat{\sigma}\right)\right)\hat{\gamma}$ and $b_1\left(m,\tau;\hat{\sigma}\right)$ respectively. In particular, $b_1\left(m,\tau;\hat{\sigma}\right)$ is the *local* optimal hedge portfolio weight under this scheme. The minimization problem under this arbitrage-based condition is defines as:

MODEL N3

$$\underset{\gamma,\sigma}{Min}\sum_{j=1}^{n}\left\{(\Delta C/C)_j-\left(1-b_1\left(m_j,\tau_j;\sigma\right)\right)\gamma-b_1\left(m_j,\tau_j;\sigma\right)\left(\frac{\Delta S}{S}\right)_j\right\}^2 K\left(\frac{\sqrt{\left(m_j-m\right)^2+\left(\tau_j-\tau\right)^2}}{h}\right)$$

. (10)

Black and Scholes assumes no riskless arbitrage opportunity, therefore, we also implement a no-arbitrage version of model N3 by fitting the Black and Scholes portfolio weight locally while setting $\gamma = r_j$ for j = 1, 2, …n in equation (10), assuming that r is a positive and constant riskless rate.

The minimization problem under no-arbitrage condition is then defined as:

MODEL N4

$$\underset{\sigma}{Min}\sum_{j=1}^{n}\left\{(\Delta C/C)_j-\left(1-b_1\left(m_j,\tau_j;\sigma\right)\right)r_j-b_1\left(m_j,\tau_j;\sigma\right)\left(\frac{\Delta S}{S}\right)_j\right\}^2 K\left(\frac{\sqrt{\left(m_j-m\right)^2+\left(\tau_j-\tau\right)^2}}{h}\right)$$

. (11)

and the local optimal hedge portfolio weight is again given by the estimator of $b_1\left(m,\tau;\hat{\sigma}\right)$.

Both local parametric estimations under arbitrage condition (model N3) and no-arbitrage condition (model N4) will be applied in this thesis to obtain the optimal delta.

Bandwidth Selection

As mentioned in Chapter 2, both least square cross-validation method and plug-in method proposed by Gasser, Kneip and Köhler (1991) will be adopted to determine the bandwidth.

Least square cross-validation method

Rudemo (1982) and Bowman (1984) propose the classical bandwidth selection method, namely least square cross-validation method. This method derives the bandwidth by minimizing a so-called cross-validation criterion and is indeed modified from integrated square error (ISE) criterion which is defined as:

$$ISE = \int_{-\infty}^{\infty} [\hat{m}_h(u) - m(u)]^2 du,$$

where $Y_i = m(X_i) + \sigma(X_i)\varepsilon_i$ for observations $\{(X_1, Y_1), \ldots, (X_n, Y_n)\}$ and $\hat{m}_h(\cdot)$ is an estimator of $m(\cdot)$ with bandwidth *h*. Expanding the quadratic term will give:

$$ISE = \int_{-\infty}^{\infty} m(u)^2 du + \int_{-\infty}^{\infty} \hat{m}_h(u)^2 du - 2\int_{-\infty}^{\infty} \hat{m}_h(u) m(u) du.$$

The unbiased estimator of the third term is $(2/n)\sum_{i=1}^{n} \hat{m}_{h,-i}(X_i)$ where $\hat{m}_{h,-i}(\cdot)$ is an estimator of $m(\cdot)$ with bandwidth *h*, without using the i^{th} observation. Since the first term is independent of the bandwidth chosen, minimizing the ISE is equivalent to minimize the least square cross-validation (LSCV) which is defined as:

$$LSCV(h) = \int_{-\infty}^{\infty} \hat{m}_h(u)^2 du - \frac{2}{n}\sum_{i=1}^{n} \hat{m}_{h,-i}(X_i).$$

In this thesis, the bandwidth selected under this scheme are be denoted by h_{CV}. In practice, h_{CV} can be obtained by:

$$h_{CV} = \arg\min_{h} = n^{-1}\sum_{i=1}^{n}\{Y_i - \hat{m}_{h,-i}(X_i)\}^2 K(X_i), \tag{12}$$

where $K(\cdot)$ is a kernel function. In this thesis, both Epanechnikov and biweight kernel functions are adopted. On the other hand, $\hat{m}_h(\cdot)$ and $\hat{m}_{h,-i}(\cdot)$ are obtained by Nadaraya-Watson kernel estimation.

Plug-in Rules

Plug-in rules exploit an asymptotic approximation to mean integrated squared error (MISE). The most common plug-in rules are those based on the assumption that the function $m(.)$ has continuous second order derivatives where b is the bandwidth, the MISE-optimal bandwidth of a boundary-adjusted Gasser-Müller estimator satisfies:

$$h = C_K\left(\frac{\sigma^2\int_0^1[f(x)]^{-1}dx}{\int_0^1\left[m_b''(x)^2\right]dx}\right)^{1/5} n^{-1/5}, \tag{13}$$

where $C_K = \left(J_K/\sigma_K{}^4\right)^{1/5}$, $J_K = \int_{-1}^{1}K^2(u)du$ and $\sigma_K^2 = \int_{-1}^{1}u^2K(u)du$.

Note that for a given K, the constant C_K is known or can be approximated arbitrarily well by numerical means. In particular, $C_K = 15^{1/5}$ and $C_K = 35^{1/5}$ for Epanechnikov and biweight kernels respectively.

The variance σ^2 may be estimated by

$$\hat{\sigma}^2 = \frac{1}{2(n-1)}\sum_{i=2}^{n}(Y_i - Y_{i-1})^2, \tag{14}$$

and $\int_0^1 [f(x)]^{-1} dx$ may be approximated by

$$n\sum_{i=1}^{n}(s_i - s_{i-1})^2, \tag{15}$$

which is motivated by the fact that $\{s_0, s_1, \ldots, s_n\}$ is a partition of $[0,1]$ and that $n(s_i - s_{i-1}) = 1/f(\tilde{x}_i)$ for $x_{i-1} \le \tilde{x}_i \le x_{i+1}$, i = 1,…,n. The most difficult quantity to estimate is $\int_0^1 [m_b''(x)]^2 dx$ because the integral depends on the unknown function $m_b''(x)$ which we do not know, or else we would not need to estimate an optimal bandwidth!

Gasser, Kneip and Köhler (GKK) (1991) provides a plug-in approach which is iterative in nature. To avoid the boundary problem when estimating $m_b''(x)$, GKK's target bandwidth is the one minimizing MISE over an interval of the form $(\delta, 1-\delta)$. (They recommend using $\delta = 0.10$). The estimated optimal bandwidth is given by

$$h_{PI} = C_K \left(\frac{0.8\hat{\sigma}^2}{\int_{0.1}^{0.9} [\hat{m}_b''(x)]^2 dx} \right)^{1/5} n^{-1/5}. \tag{16}$$

A possible estimator $\hat{m}_b''(x)$ is a Gasser-Müller kernel type estimator

$$\hat{m}_b''(x) = b^{-3} \sum_{i=1}^{n} Y_i \int_{s_{i-1}}^{s_i} K\left(\frac{x-u}{b}\right) du, \tag{17}$$

where K is a kernel with support (–1,1) and the s_i's are as defined before and satisfies the moment conditions:

$$\int_{-1}^{1} u^k K(u) du = 0,\ k = 0, 1, \text{ and } \int_{-1}^{1} u^2 K(u) du = 2.$$

The conditions above are to ensure the asymptotic unbiasedness of $\hat{m}_b''(x)$.

Assuming evenly spaced design points, GKK proposes an iterative algorithm as follows:

- Take $\widetilde{h}_0 = 1/n$.

- Define $\tilde{h}_i = C_K \left(\dfrac{\hat{\sigma}^2(1-2\delta)}{\int_\delta^{1-\delta} [\hat{m}_b''(x)]^2 dx} \right)^{1/5} n^{-1/5}$, $b = \widetilde{h}_{i-1} n^{1/10}$ for $i = 1, \ldots, 11$.

- Use $\widetilde{h}_{11}$ as the plug-in bandwidth.

According to GKK, the motivation for the factor $n^{1/10}$ and the use of eleven iterations is based on asymptotic considerations.

CHAPTER FOUR

DATA DESCRIPTION

To compare the empirical relevance of the parametric and nonparametric estimations of optimal delta, we apply the approaches to the hedging of the Nikkei 225 index and its options. In this section, we will first present the backgrounds and the characteristics of the data.

Nikkei 225 Index

The First Section of Tokyo Stock Exchange (TSE) is the most important stock exchange in Japan, which accounts for about 86% of the trading value and volume on all stock exchanges there. Meanwhile, 225 stocks that are traded in the First Section of Tokyo Stock Exchange (TSE) make up the Nikkei 225 index. This index has the longest history in Japanese stock markets and has been introduced since September 1950. Now, it is computed and published by Nihon Keizai Shhimbun, Inc., a leading Japanese financial information services firm.

The Nikkei 225 index is a simple, equally-weighted average similar to the Dow Jones Industrials: the sum of the prices of the constituent shares is scaled and adjusted for stock splits with a divisor factor. As given by its name, the index covers the stocks of 225 companies from all sectors of Japanese economy including industrials, trading companies, and financial institutions. The index has been calculated and announced minute-to-minute since October 1985. The component issues of this index have been updated annually since October 1991 so as to enhance its liquidity. Moreover, 225 constituent stocks of Nikkei 225 index account for about 50% of the overall capitalization of the First Section. The index is therefore capable of reflecting the changes in the market environment and maintaining the overall consistence of the stock markets in Japan.

Undoubtedly, Japan has been playing an important role in the world economy since 1980. The Japanese stock markets are comparable to their counterpart in many other developed countries in terms of stock market capitalization and hence are significant to international investors. Therefore, the Nikkei 225 index becomes the most frequently cited index for the Japanese stock markets.

Options on Nikkei 225 index

The driving forces of the bullishness in Japanese stock markets during the latter half of 1980s were domestic institutional investors. In order to meet their eager demand for financial instruments to hedge their huge amount of stock holdings against possible declines, derivative securities on stock indices were introduced soon after the revision of Japanese Security Transaction Law in 1988.

From very beginnings in little noticed over-the-counter markets, Japanese securities markets were slow to develop derivative instruments such as futures and options based on stock market indices or on the value of individual stocks. Currently, there are three stock index options in Japan, namely, TOPIX (Tokyo stock price index) option, Nikkei stock index option, and option 25 stock option. They are listed on the Tokyo, Osaka, and Nagoya stock exchanges, respectively in 1989.

Specifically, we examine the Nikkei stock index option, since this market seems to be the most efficient among the three in terms of its dominant transaction value. It is often said that the Nikkei stock index option market, being the first option market in Japan, has gained its entrepreneur's profit, as the S&P 100 index option in the United States has. The manageability of underlying stock indices and the timing of introduction are the main reasons of the Nikkei stock index option's popularity.

The Osaka Security Exchange (OSE) introduced American style call and put options on the Nikkei 225 index on June 12, 1989. Nikkei 225 option trading is now expanded to oversea markets. On September 25, 1990, the Chicago Mercantile Exchange (CME) inaugurated index option trading based on the Nikkei 225. Such American style options gain much popularity because they offer the holders the right to exercise the options at any time to maturity. Nikkei 225 options expire on the tenth day of the expiration month. There are four contract months and five different strike prices available in a given month, with strike price differing by 500 index points. Each option contract is worth 1000 times its price quoted in the index points. On June 12, 1997, for example, the Nikkei index closed at 13,398 and a July call with an exercise price of 13,000 sold for 895. The option's actual purchase price was 895 times 1000 or 895,000 yen. If exercise immediately, the owner would have received 1000 times the difference between the Nikkei index and the strike price, (13,398.01-13,000) $\times$ 1000 or 398,000 yen.

The Nikkei 225 index option is designed as an American dividend paying option. However, before determining what model is used in estimation of the optimal hedge ratios, the following two factors are noteworthy: First, unlike those in the United States, dividend ratios of most Japanese corporate stocks are significantly low so that dividend payment can be ignored without affecting empirical results crucially. The weighted-average dividend yield for 225 stocks covered by the Nikkei 225 index is 0.75%, as of the end of August 1990. This figure is much smaller than 3.78% for S&P

500 issues, as of the corresponding day. Second, the Nikkei stock index option can be currently exercised only on Thursdays and on the expiration day, rather than at any time as is the case for true American options. Geske and Johnson (1984) shows that the difference in value is likely to be insignificant. In addition, the size of early exercise has been insignificant so far. Therefore, the Nikkei 225 index option used in this paper can be regarded as European non-dividend paying option that is suitable for the Black and Scholes model.

Data Source

Daily settlement prices of Nikkei 225 index, the yields of 1-month Euroyen contracts and 2-month Gensaki (repo) contracts from June 12, 1989 to July 14, 2000 are collected from the Datastreams International. On the other hand, daily data of Nikkei 225 index call options within the same period are supplied by Osaka Security Exchanges. The option database consists of contract month code specifications, trading date, opening price, high price, low price, closing price, trading volume, open interest and trading value.

Basically, the daily closing prices of the Nikkei 225 index call options are used. However, a problem may arise when the last option trade of any issue occurs far before the closing time of the underlying market. As for the issues with very thin trading volume, the last trade in a day could occur even in the morning. In such case, a significant time-lag effect can be serious.

Nevertheless, the daily closing prices have been used since intra-day transaction data are not available, and the results are expected to be meaningful because errors are likely to offset one another in a large number of sample data.

Data Compilation

We divide the option data into several classes according to their moneyness and the term to expiration. By definition, the ratio between the Nikkei index and the option exercise price $(S(t)/X)$ is the time-*t* moneyness of an option. A Nikkei 225 index call option is then said to be in-the-money (ITM) if $S(t)/X \geq 1.05$, at-the-money (ATM) if $S(t)/X = 1$[2] and out-the-money (OTM) if $S(t)/X \leq 0.95$. For the time-to-maturity (τ), a Nikkei 225 index option can be classified as a short-term one if $\tau \leq 1/12$, a medium-term one if $1/12 < \tau < 2/12$ and a long-term one if $\tau \geq 2/12$. We only focus on short-term and long-term options in this study because a medium-term option series is difficult to generate in practice. The proposed moneyness and maturity classifications procedure produces 6 series for which the empirical results will be reported. A single nonoverlapping sequence of options for each series is generated by choosing the initial moneyness and time-to-maturity that is closest to the value stated above. As option trading is typical in all markets including Japan, Baring Securities (1990) shows that most of the trading volume concentrates in short maturity options with strike prices near the current value of the Nikkei index.

Moreover, in order to lessen the effect of non-synchronous trading and liquidity problem, we do not consider those option prices with contract volume smaller than 3. Furthermore, call option prices that do not satisfy the lower bound condition $S(t) - Xe^{-r\tau}$ are excluded, where $S(t), X, \tau, r$ are the underlying Nikkei 225 index price, the strike price, the time-to-maturity (in years), and the riskless interest rate correspondingly.

[2] In practice, we choose the initial moneyness for at-the-money series between 0.95 and 1.05 interval, which is closest to 1.

We approximate the riskless interest rates as the yields of 1-month Euroyen contracts for short-maturity option and the yields of 2-month Gensaki (repo) contracts for long-maturity option on the close of the month before the initial activity in that option. Furthermore, we estimate the Black and Scholes historical volatility for Nikkei 225 index using its continuously compounded daily returns on the close of the month before the initial activity in that option.

CHAPTER FIVE
EMPIRICAL FINDINGS

Estimation Results

Parametric Model

Black and Scholes Model

The estimation results are reported in Figures 1 to 6. For Black and Scholes model (model BS), the closed-form delta (equation 2) can be directly interpreted as the optimal hedge ratios for different moneyness and time-to-maturity categories. The estimated deltas for all categories are all positive and less than one. This suggests that using delta hedging for a short (long) position in a European call option involves keeping a long (short) position of $N(d_1)$ shares at any given time. For each series, we observe that the variations of deltas are generally similar to that of the spot price. One would also discover that the variation of deltas is almost identical to that of the spot price for in-the-money options.

We also record several typical patterns for the variations of deltas with time-to-maturity for options with different levels of moneyness. Focusing on the last maturity month of our sample (June 2000), Figures 7 and 8 show the variations of deltas with time-to-maturity with different levels of moneyness for short-term and long-term options respectively. Results show that deltas of both out-the-money and at-the-money series increase with time-to-maturity, whereas deltas of in-the-money series decrease with time-to-maturity.

Nonparametric Models

Local Polynomial Estimation

In this thesis, local linear and local quadratic models are estimated to determine the optimal delta. Prior to any estimation, the least squares cross-validation method and plug-in rule are used to obtain the optimal bandwidths for each series. For the least squares cross-validation method, we apply the NW kernel estimator to estimate $\hat{m}_{h,-i}(\bullet)$ in equation (12), therefore the h_{CV}'s are the same for both local linear and local quadratic models. Similarly, for the plug-in rule, we use the GM kernel estimator to estimate $\hat{m}_b''(\bullet)$, hence the h_{PI}'s are the same for both local linear and local quadratic models. The resulting bandwidths are reported in Table 4.

It should be noted that for all series (estimated under both Epanechnikov and biweight kernels), h_{CV}'s are smaller than h_{PI}'s. Smaller bandwidth implies that the resulting estimates will have smaller biases but larger variances. In addition, with the exception of the long-term and at-the-money series, the estimated bandwidths (both h_{CV}'s and h_{PI}'s) generally increase with time-to-maturity. This may be due to the fact that smaller number of observations is used for estimation as time-to-maturity increases and the data become sparser and more thinly scattered. The schemes select a larger bandwidth to reduce the variance and hence to obtain a smoother estimation. Yet, we cannot draw a similar conclusion for the variation of bandwidth with moneyness.

For each series, the range of the independent variable $(\Delta S/S)_j$ will be divided into 1000 grids points and equations (6) and (7) are estimated at each of these grid points for models N1 and N2 respectively. The resulting deltas (estimated under

Epanechnikov kernel) are shown in Figures 9 to 32.

For all series, the estimated deltas are more variable if the bandwidths are selected under the least square cross-validation method. This is the same for both local linear and local quadratic models and is the direct effect of using a smaller bandwidth in estimation. As we discussed before, the resulting estimates will be very bumpy for smaller bandwidth. Moreover, some illogical estimates would be obtained under this bandwidth selection method. For example, for the short-term and out-the-money series, some estimated deltas are too large in magnitude (e.g., greater than 2). These absurd estimates usually appear in the regions where the observations are thinly scattered.

With either bandwidth selection method, we find that the resulting estimates for short-term options and those for long-term options are generally similar in pattern across different levels of moneyness. However, the estimated deltas for short-term options are indeed more variable, especially when least square cross-validation method is used to select the bandwidths. In fact, this is also the direct effect of using a smaller bandwidth for short-term options in estimation.

On the other hand, by comparing the estimated deltas between those from model N1 and those from model N2 with bandwidths selected under the least square cross-validation method, we find that the patterns of fluctuations look very alike. In general, the estimated deltas are highly variable at extreme values of $(\Delta S/S)_j$'s. Again, it is due to the sparse distribution of observations in these regions.

When the plug-in rule is used to select the bandwidth, the resulting estimates from model N1 and those from N2 also have a similar pattern. Furthermore, for each estimated model, the estimated deltas now become a much smoother function of $(\Delta S/S)_j$'s. It is simply because the bandwidths used are larger under the plug-in rule. The resulting function under local quadratic model (model N2) seems to be an exaggerated version of that under local linear model (model N1). Again, it is particularly true for the extreme values of $(\Delta S/S)_j$'s. It should then be emphasized that although estimation with this bandwidth selection scheme appears to be quite different from that with the least square cross-validation method, they both have similar problem near the extreme values.

Local Parametric Estimation

Apart from the local polynomial estimation, we also use local parametric estimation (by fitting Black and Scholes model locally) to determine the optimal delta. Similar to the case of local polynomial estimation, both least squares cross-validation method and plug-in rule are used to obtain the optimal bandwidths for each series. The resulting bandwidths are reported in Table 4.

Analogous to the previous estimation, h_{CV}'s are much smaller than h_{PI}'s for all series (estimated under both Epanechnikov and Biweight kernel) while the estimated bandwidths (both h_{CV}'s and h_{PI}'s) increase with time-to-maturity. In additional, with the exception of the at-the-money series, the estimated bandwidths (both h_{CV}'s and h_{PI}'s) generally increase with moneyness.

For each series, the range of the independent variables (m_j, τ_j) will be divided into 1000 grids points and equations (10) and (11) are estimated at each of these grid points for models N3 and N4 respectively. Focusing on the last maturity month of our sample (June 2000), the variations of resulting deltas (estimated under Epanechnikov kernel) with time-to-maturity for all series are displayed in Figures 33 to 38. Results show that deltas of both out-the-money and at-the-money series generally increase with time-to-maturity whereas deltas of in-the-money series decrease with time-to-maturity.

We also observe that the estimated deltas are more variable if the bandwidths are selected under the least square cross-validation method. This is the same as the case of local polynomial estimation and is the direct effect of using a smaller bandwidth in estimation.

With either bandwidth selection method, deltas for short-term and long-term options are generally similar in pattern across different moneyness levels. Yet, the estimated deltas for long-term options are indeed more variable, especially when bandwidths are selected under the least square cross-validation method. On the other hand, we find that the variability of deltas, however, increases with moneyness for short-term options. Nevertheless, this pattern of fluctuations disappears for long-term options.

By comparing the estimated deltas between those from model N3 and those from model N4, we find that the patterns of fluctuations look very alike, especially when plug-in rule is used to select the bandwidths. Moreover, the resulting function under model N3 seems to be an exaggerated version of that under model N1.

Evaluation of Model Performance

This section presents the approaches which are applied to evaluate the performance of various models in the estimation of the optimal delta.

Performance Measures

The ultimate goal of various models is to estimate the optimal delta which can minimize the risk exposure of a particular position in the option market. One meaningful measure of performance for a given model is the "tracking error" of various replicating portfolios designed to delta-hedge an option position, using the estimated deltas. In particular, suppose at date 0 we sell one call option and undertake the usual dynamic trading strategy in stocks and bonds to hedge this call during its life. If we have correctly identified the model, and if we can costlessly and continuously hedge the call, then at expiration the combined value of our stock and bond positions should exactly offset the value of the call. The difference between the terminal value of the call and the terminal combined value of stock and bond positions may then serve as a measure of the accuracy of our approaches.

Formally, we denote $V(t)$ as the dollar value of our replicating portfolio at date t and let

$$V(t) = V_S(t) + V_B(t) + V_C(t); \tag{18}$$

where $V_S(t)$ is the dollar value of stocks, $V_B(t)$ is the dollar value of bonds, and $V_C(t)$ is the dollar value of call options held in the portfolio at date t. The initial composition of this portfolio at date 0 is assumed to be:

$$V_S(0) = S(0)\Delta_i(0); \tag{19}$$

$$V_C(0) = -C(0); \tag{20}$$

$$V_B(0) = -(V_S(0) + V_C(0)); \tag{21}$$

where $C(\bullet)$ is the actual call price.[3] The portfolio positions in equations (19) to (21) represent the sale of one call option at date 0, priced according to the actual call price in the option market, and the simultaneous purchase of $\Delta_i(0)$ shares of stock at price $S(0)$, where $\Delta_i(0)$ is the estimated delta by model i, and i = BS, N1, N2, N3, N4, HALF and SWITCH. Since the stock purchase is wholly financed by the combination of riskless borrowing and proceeds from the sale of the call option, the initial value of the replicating portfolio is identically zero, and thus

$$V(0) = V_S(0) + V_B(0) + V_C(0) = 0. \tag{22}$$

Prior to expiration, with a discrete interval of length τ, the stock and bond positions in the replicating portfolio will be rebalanced so as to satisfy the following relations:

$$V_S(t) = S(t)\Delta_i(t); \tag{23}$$

$$V_B(t) = e^{r\tau}V_B(t-\tau) - S(t)(\Delta_i(t) - \Delta_i(t-\tau)). \tag{24}$$

The tracking error of the replicating portfolio is then defined to be the value of the replicating portfolio $V(T)$ at expiration date T. Hence, we can obtain the following performance measure:

$$V(T) = V_S(T) + V_B(T) + V_C(T); \tag{25}$$

[3] Hutchinson, Lo and Poggio (1994) estimate the tracking error by using the theoretical Black and Scholes call price. We try this approach, however, results are far inferior. Therefore, we decide not to report any results from this approach.

with

$$V_S(T) = S(T)\Delta_i(T); \tag{26}$$

$$V_B(T) = e^{r\tau}V_B(T-\tau) - S(T)(\Delta_i(T) - \Delta_i(T-\tau)); \tag{27}$$

$$V_C(T) = -(S(T) - X); \tag{28}$$

where X is the exercise price of the call option.

With the knowledge of tracking error, we can obtain the mean tracking errors to compare the hedge performance across different models. Before any evaluation, we soon observe a bias in the tracking error of locally estimated replicating portfolios. In particular, the mean tracking error is almost invariably positive when using locally estimated portfolio weights. With Black and Scholes hedge ratios, the mean tracking error is much smaller. To understand the impact of this bias, we also report the standard deviations of the tracking errors. In this study, we look at the (signed) tracking errors, the absolute tracking errors and the squared tracking errors. We also report the frequency (in %) that the former two measures outperform Black and Scholes model while the freqency of the latter measure is not display here because it is the same as that of the absolute tracking errors.

One could attribute the bias in the tracking error of locally estimated hedges to the well-known biases of local estimation when an optimal bandwidth is selected. This conjecture is proven to be wrong in the following manner. Biases in local estimation decrease as the bandwidth is lowered. Hence, we ought to observe decrease in bias of the tracking error of locally estimated hedge portfolios as the bandwidth is reduced. Instead, we record no such changes, and therefore reject the conjecture. As we will argue later, the average positive return on the hedge portfolio may reflect the mispricing of options in the world where one can only hedge in discrete time.

Gouriéroux and Laurent (1994) reports that the average correlation between the tracking errors of locally fitted hedge portfolio weights and Black and Scholes portfolio weights is found to be surprisingly low. Hence, we also investigate the tracking performance of a portfolio whose hedge ratio is obtained by an equally weighted average of the locally fitted ratio and the Black and Scholes ratio. As we will see, this improves on either way of obtaining the hedge ratios. In other words, it is preferable to combine both procedures. This finding essentially means that local parametric estimated hedge ratios are based on different information, unlike that used in computation of the historical volatility. The latter is used to compute the Black and Scholes hedge ratio. Consequently, the optimal strategy from a decision-theoretical point of view uses a combination of traditional option pricing and local parametric derivative analysis.

One could also conjecture that the improved performance of the combined hedge strategy is due to the superiority of nonparametric analysis for hedging of out-the-money options, while this superiority disappears as the derivative moves in-the-money. To evaluate this possibility, we also investigate the hedging performance of a strategy whereby we switch from locally estimated hedge portfolio weights to Black and Scholes weights as the moneyness increases above 1.05. However, it will be shown that this strategy is dominated even by the one where locally estimated hedges are used throughout. Consequently, this alternative explanation of the impressive performance of the combined strategy can be proven to be false.

Let us now turn to a discussion of the results. Tables 5 to 12 display the in-sample hedging performance whereas Tables 13 to 20 display the out-of-sample hedging performances where the hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under either arbitrage or no arbitrage conditions (models N3 and N4 respectively), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH). We examine model SWITCH simply because hedging here is to track one given option until its maturity, the target option is then likely to move in- or out-the-money over its life span. Results are reported for initial moneyness equalling to 0.95 (out-the-money), 1.00 (at-the-money) and 1.05 (in-the-money) and for time-to-maturity equalling to one month (short-term) and two months (long-term).

In-sample Performance Evaluation

The sample used for estimation ranging from June 12, 1989 to December 31, 1999. Both locally fitted models, local polynomial and local parametric models, are compared with Black and Scholes model (a parametric model) that is used as a benchmark.

Table 5 shows the in-sample hedging performance for short-term options. We specify the first order polynomial, second order polynomial, local parametric approach under arbitrage condition and Epanechnikov kernel function to control the weighting scheme. The most crucial component, the bandwidth h, is determine by the least square cross-validation (CV) method and plug-in rule (PI) correspondingly. We first evaluate the tracking error, as already pointed out before, the average returns of the hedge portfolio by all estimation methods are positive. However, we cannot find a nonparametric model which can consistently outperform the others in all series.

Yet except model N1 (PI), we observe the performance of local polynomial estimates for all levels of moneyness are far inferior to local parametric estimates in terms of both absolute and squared tracking errors. This indicates that parametric models such as Black and Scholes model provide useful information about the local curvature of the replicating portfolio weights as a function of moneyness and maturity.

The improvement of local parametric model, model N3, over model BS is pronounced for out-the-money options. The superiority is clearest in terms of root mean squared tracking error for initial moneyness of 0.95. Model N3 (CV) provides a 12.15% improvement.[4] This means that, in some cases, the quadratic loss functions, which prefer to penalize outliers heavily, would be especially attracted by our technique. However, this superiority reduces to 8.68% for at-the-money options ($m = 1.00$) and eventually no improvement for in-the-money options ($m = 1.05$) whereas the frequency that model N3 (CV) outperforms model BS drops from approximately 40% to 32% and eventually 26%.

The reduction in performance of local parametric estimation as a function of the initial moneyness of the option is not surprising. As noted in Bossaerts and Hillion

[4] The percentage improvement is calculated by (1162.79-1323.6)/1323.6. Percentage improvements to be reported below are computed analogously.

(1997), for in-the-money options, the relationship between call returns and stock returns is essentially linear. Black and Scholes hedges assume linearity whereas local parametric model is designed to capture nonlinearities. Therefore, it can be expected not to outperform when the true relationship is linear.

It should be noted that the combination of Black and Scholes hedges with locally estimated portfolio weights generates interesting results. Model HALF3 enhances the performance for all levels of moneyness in terms of both absolute and squared tracking errors. Even for in-the-money options ($m = 1.05$), models HALF1 (PI), HALF3 (CV) and HALF3 (PI) still provide a 24.73%, 26.74% and 14.73% improvement respectively in terms of root mean squared tracking error. We will see a promising track record of such combined policy in the out-of-sample forecast and explain the intuition behind.

To shed light on the importance of the Black and Scholes model in options analysis, we unintentionally disable local polynomial technique in terms of its ability to properly describe the target function. We, therefore, conclude from Table 5 that local polynomial models are far inferior to local parametric models for all levels of moneyness while the superiority of local parametric models, especially when bandwidths are selected by least square cross-validation method, decreases with moneyness.

Table 6 shows the in-sample hedging performance for long-term options. Compared with Table 5, similar results are obtained across different levels of moneyness. However, we find that the superiority of local parametric model at all levels of moneyness reduces a lot. Model N3 provides no improvement in terms of absolute tracking error whereas, in terms of root mean squared tracking error, model N3 (CV) only provides a 3.11% and 3.63% improvement for initial moneyness of 0.95 and 1.00 respectively. Bossaerts and Hillion (1997) suggests that the relationship between call returns and stock returns for long-term options is essentially linear while local parametric model is designed to capture nonlinearities. Therefore, it should not be expected to outperform Black and Scholes model.

Yet, we also observe that the dominance of combined policy reduces a lot. For in-the-money options ($m = 1.05$), models HALF1 (PI), HALF3 (CV) and HALF3 (PI) only provide a 15.93%, 8.07% and 8.07% improvement respectively in terms of root mean squared tracking error. Obviously, the superiority of local parametric models decreases with maturity while local polynomial model still under-performs Black and Scholes model.

Tables 7 and 8 show the in-sample hedging performances for short-term and long-term options respectively. Unlike Tables 5 and 6, we specify the local parametric approach under no-arbitrage condition. This means that we implement local parametric estimation with $\gamma = r$ in equation (10). Results are analogous to Tables 5 and 6. We conclude that local polynomial models under-perform local parametric model (model N4) while local parametric model works well for both short-term and out-the-money options.

It should also be emphasized that the superiority of local parametric model under no-arbitrage condition (model N4) is inferior to the less restrictive one that estimated under arbitrage condition (model N3). We record that the dominance of model N4 reduces for all series in terms of both absolute and squared tracking errors. Moreover, in terms of root mean squared tracking error for initial moneyness of 0.95, model N4 (CV) only provides a 7.93% improvement for short-term options and 0.55% improvement for long-term options which is 4.22% and 2.56% less than model N3 (CV) respectively.

To gauge the irrelevance of kernel function used in our study, we repeat the above estimations with biweight kernel function. The corresponding in-sample hedging performances are displayed in Tables 9 to 12. Compared with Tables 5 to 8, similar results are concluded. This indicates that, as noted in Fan and Gijbels (1996), the choice of kernel function is not essential to the performance of the resulting estimators, both theoretically and empirically. However, we notify that the superiority of model HALF1 (PI) for short-term and long-term in-the-money options disappears in terms of both absolute and squared tracking errors.

Out-of-sample Forecast Evaluation

In order to obtain a better understanding of how the nonparametric models perform relative to our benchmark, Black and Scholes model, in this section we will evaluate the out-of-sample performance of the models and see if the conclusions drawn above are still valid.

Hedge Portfolio Weights for Out-of-sample Observations

To obtain the hedge portfolio weights for the out-of-sample period, one can carry out one-step ahead forecast. For example, let N be the number of in-sample observations. Then in order to forecast the hedge portfolio weights for the $N+i^{th}$ observation (i.e., i period(s) ahead), one has to use the $N + i - 1$ observations to estimate the models. Therefore, if there are M out-of-sample observations, we have to estimate each model M–1 times for one-step ahead forecast. This approach is optimal in a sense that it makes use of the maximum amount of information available. However, if M is not large (relative to N), the incorporation of out-of-sample observations in the estimation may only have little impact on the results.

Since the number of out-of-sample observations over the period of January 1, 2000 to July 14, 2000 is small relative to that of in-sample observations (about one-tenth), we do not carry out one-step ahead forecast. Rather, we use the coefficient estimated with in-sample data directly to obtain the out-of-sample hedge portfolio weights. For local polynomial estimation, given $(\Delta S/S)_{N+i}$ (i.e., i period(s) ahead), we use the hedge portfolio weight (estimated with in-sample data) at grid point which is nearest to $(\Delta S/S)_{N+i}$. For local parametric estimation, given (m_{N+i}, τ_{N+i}) (i.e., i period(s) ahead), we use the hedge portfolio weight (estimated with in-sample data) at grid point which is nearest to (m_{N+i}, τ_{N+i}).

Evaluation for Out-of-sample Performance

Methods employed for in-sample evaluation are also applied to evaluate out-of-sample performance. Our results are very similar to Bossaerts and Hillion (1997). Table 13 shows the out-of-sample hedging performance for short-term options. The specifications are the same as Table 5. First, we evaluate the tracking error. The average returns of the hedge portfolios are not invariably positive. Again, we cannot find a nonparametric model which can consistently outperform the others in all series.

The performance of local polynomial models for all levels of moneyness is still inferior to local parametric models in terms of both absolute and squared tracking errors. As mentioned, this indicates that parametric models such as Black and Scholes provide a useful information for local volatility estimation (used, for instance, in Rubinstein, 1994). In fact, local volatility is a crucial parameter that absorbs the misspecification of using Black and Scholes hedge portfolio weights in discrete time, and, hence, it will generally depend on moneyness and maturity. On the contrary, we record that model N1 (PI) consistently beats model BS for all levels of moneyness in terms of squared tracking error, and, especially, it provides a 24.41% improvement for in-the-money options.

The superiority of local parametric model, model N3, over model BS is pronounced for out-the-money options. In terms of root mean squared tracking error for initial moneyness of 0.95, Model N3 (CV) provides a 18.39% improvement. However, this superiority reduces to 9.75% for at-the-money options ($m = 1.00$) and eventually no improvement for in-the-money options ($m = 1.05$). Similar to the in-sample evaluation, the reduction in performance of local parametric model as a function of the initial moneyness of the option is still observed.

By far the most impressive performance is generated by the combination of hedges based on locally fitted models and Black and Scholes model. As mentioned in Bossaerts and Hillion (1997), this is due to the less-than-perfect correlation between the tracking errors of locally fitted models and Black and Scholes model. For out- and at-the-money options, we record that all models HALF's, except model HALF2 (PI), outperform model BS in terms of both absolute and squared tracking errors. Even for in-the-money options, models HALF1 (PI), HALF2 (PI), HALF3 (CV) and HALF3 (PI) still provide a 60.18%, 24.85%, 15.20% and 13.67% improvement respectively in terms of root mean squared tracking error.

The low correlation between the hedge error using locally fitted estimation and that from Black and Scholes reflects of the low correlation between the sampling errors of the statistics behind each methodology. Black and Scholes hedges use the historical volatility as the main statistical input; while locally fitted estimation exploits the correlation between call and stock returns. It appears that the errors of estimation of historical volatilities and correlations are not perfectly correlated. Hence, an improvement in the out-of-sample hedging performance is obtained by combination both procedures.

The promising track record of the combination of Black and Scholes hedges and locally estimated portfolio weights cannot be attributed to the former's enhanced performance for in-the-money options. If this is the case, a policy whereby one switches from locally fitted analysis to Black and Scholes from the moment the option's moneyness reaches 1.05 would do much better. In fact, Table 13 documents that such a policy is inferior across the board.

We conclude from Table 13 that local polynomial models, except local linear model with bandwidth selected under plug-in rule, are still inferior to local parametric models for all levels of moneyness while the superiority of local parametric models, especially when bandwidths are selected by least square cross-validation method, decreases with the levels of moneyness. Besides, local linear model with bandwidth selected under plug-in rule is the one and the only one model that beats model BS for short-term and in-the-money options.

Table 14 presents the out-of-sample hedging performance for long-term options. Compared with Table 13, similar results are obtained across different levels of moneyness. Nevertheless, we find that the superiority of local parametric model, except model N3 (CV), disappears for all levels of moneyness. Only model N3 (CV) provides a 6.25% improvement in terms of squared tracking error for out-the-money options. As discussed before, this indicates that the superiority of local parametric models decreases with maturity.

On the contrary, we obtain a remarkable out-of-sample hedging performance of local polynomial estimation for long-term options. By comparing the outperformance of various models, we find that local polynomial model keeps its lead over local parametric model for all levels of moneyness. In terms of squared tracking error, model N2 (PI) provides a 45.77% improvement for out-the-money options, which is 39.53% higher than model N3 (CV) while model N1 (PI) provides a 50.27% improvement for at-the-money options. Apparently, all local polynomial models, except model N2 (CV), outperform both models BS and N3 for in-the-money options in terms of both absolute and squared tracking errors. In particular, model N1 (PI) provides a 81.94% improvement in terms of root mean squared tracking error. As also noted in Table 13, this implies that local polynomial models, especially model N1 (PI), generate an extraordinary outperformance for in-the-money options.

The promising track record of the combined policy andthe inferior result of the switch policy imply that an improvement in the out-of-sample hedging performance is obtained by combination of both locally fitted models and Black and Scholes model. To conclude, clearly the superiority of local parametric models decreases with maturity. Local polynomial models, especially local linear model with bandwidth selected under plug-in rule, generate a surprising outperformance over both Black and Scholes and local parametric models for long-term, and, especially in-the-money options.

Tables 15 and 16 show the out-of-sample hedging performances for short-term and long-term options respectively. Similar to Tables 7 and 8, we specify the local parametric approach under no-arbitrage condition (model N4). Results are very similar to Tables 13 and 14. Focusing on the performance of model N4 (CV) for short-term and out-the-money options, we record a 17.15% improvement in terms of root mean squared tracking error, which is just 1.24% less than the less restrictive one that is estimated under arbitrage condition (model N3). Hence, we conclude that the implementation of arbitrage or no-arbitrage conditions for out-of-sample forecast should not make a large difference.

To gauge the irrelevance of kernel function used in our out-of-sample forecast, we repeat the above estimations with biweight kernel function. The corresponding out-of-sample hedging performances are displayed in Tables 17 to 20. Compared with Tables 13 to 16, we find that the hedging performance estimated under biweight kernel function is at least as good as Epanechnikov kernel function. As mentioned before, results are consistent with Fan and Gijbels (1996).

CHAPTER SIX
CONCLUSION

Since the seminal paper by Black and Scholes (1973), many studies have focused on how options should be priced and what the optimal option-hedging strategies are. The basic function of option is hedging. Hence, there is indeed a genuine need for market-markers and hedgers to determine the optimal hedge ratio, which is usually referred to as delta, to minimize their risk on a particular position in the options market.

This thesis aims at estimating such optimal delta by employing various econometric methods, namely, the local polynomial estimation and local parametric estimation. These estimation techniques belong to the family of nonparametric models and are compared with our benchmark parametric model, the Black and Scholes model. These models are applied to the Nikkei 225 index and its options traded in Osaka Security Exchange. Their performance is then evaluated in terms of "tracking error" across different levels of moneyness and time-to-maturity.

The benchmark model used in our study is the Black and Scholes model, from which the closed-form delta based on many restrictive and unrealistic assumptions is formulated. Focusing on the continuous-time modelling, one can immediately mention the impossibility of hedging in continuous time. When continuous-time portfolio weights are applied to a discrete-time hedging problem, the misspecification of the Black and Scholes model will lead to the discretization-induced tracking error. In fact, this traditional model belongs to the family of parametric models and assumes particular functional forms for the underlying variables. This model is simple and direct but its rigidity induced many inadequacies especially if the assumed functional form is inappropriate.

On the other hand, nonparametric models do not imposed any functional form for the underlying quantities and "let the data speak for themselves". Among numerous nonparametric estimation techniques, we employ local polynomial (local linear and local quadratic) estimation and local parametric (under arbitrage and no-arbitrage conditions) estimation to obtain the optimal deltas. As discussed before, one important input which is crucial to any nonparametric model is the bandwidth. It determines how large the local neighborhood should be in estimation. In this thesis, two automatic and data-driven bandwidth selection methods are used. These methods include the least square cross-validation method and the plug-in rule. Results show that, in general, bandwidths selected under the former method are much smaller than those selected under the latter, and the resulting estimates of the hedge ratios are much more variable with the former method.

The estimated deltas under different models are evaluated by comparing the "tracking error", which is defined as the maturity-date dollar payoff on a self-financing portfolio that long one call option and short the replicating portfolio. The smaller the tracking error is, the better the hedge will be.

For the in-sample data series, we find that local polynomial models, except local linear model with bandwidth selected under plug-in rule, are far inferior to local parametric models due to the exclusion of information such as moneyness and maturity in this technique. On the contrary, local parametric models with bandwidth selected under the least square cross-validation method are superior to the Black and

Scholes model. Yet, the superiority of local parametric estimations decreases with both moneyness and maturity. Unlike local polynomial estimation, local parametric estimation combines the power of analytical derivative securities analysis with the flexibility of locally weighted averaging. The result is therefore a technique that generates significant improvements in hedging performance in situations where the theoretical option pricing model is suspected to be inadequate, such as in discrete-time hedging for option whose return is nonlinearly related to that of the underlying securities, namely, short-maturity and out-the-money options. From the results, we can also conclude that the performance of nonparametric estimation techniques depends crucially on the method used to compute the bandwidth.

For the out-of-sample data series, local parametric models especially when bandwidths are selected by least square cross-validation method, still beat the Black and Scholes model. Nevertheless, local polynomial models, especially local linear model with bandwidth selected under plug-in rule, generate surprising outperformance over the Black and Scholes model particularly for long-term and in-the-money options. In addition, we find that best out-of-sample tracking records are obtained by combining locally estimated hedge portfolio weights with Black and Scholes. The optimality of this combination indicates that our locally fitting approach to options analysis and traditional derivatives analysis exploit complementary information from historical samples.

In fact, option hedging is a dynamic process. The optimal hedge ratio, delta, is then of great interest to every investors in maintaining their delta-neutral positions. Based on the above results, we would like to conclude this thesis by making some comments on the issue of whether the Black and Scholes model could provide an effective benchmark for delta estimation on the Japanese representative stock index.

First, a glance on the plots of deltas from local parametric models (figures 33 to 38) may lead to an impression that the resulting deltas from local parametric models are far less than the Black and Scholes deltas, especially for short-maturity and out-the-money options. This implies that the Japan stock market has a tendency to hedge less than what Black and Scholes suggested and, therefore, investors are indeed not risk-neutral but risk-loving in general.

Second, investors should pay close attention to the method of estimating volatilities since they form the crucial input with which to compute deltas. Results of this study indicate that delta estimation using the Black and Scholes model based on historical volatility is indeed sufficient. However, local parametric approaches based on implied volatilities are superior to the Black and Scholes model in many cases and can bring significant improvement in hedge performance.

Third, as also noted in Bossaerts and Hillion (1997), the errors from Black and Scholes hedging in discrete time are most important for short-maturity and out-the-money options. Thus, investors should better not use the Black and Scholes estimated deltas of these options as benchmarks in actual option hedging.

Finally, the promising track record of the combination of Black and Scholes hedges with locally estimated portfolio weights suggests that such combined option hedging policy is the optimal strategy from a decision-theoretical point of view.

Table 1: Comparison of Minimax Efficiency among Linear Estimators

Kernel Function	Local Linear	Gasser-Müller	Nadaraya-Watson
Epanechnikov	100	66.67	0
Gaussian	95.12	63.41	0
Uniform	92.95	61.97	0

Source: Fan and Gijbels (1996), p 86.

Table 2: MISE's from Different Kernel Functions

Kernel Function	γ	Form	Asymptotic MISE
Epanechnikov	1	$\frac{3}{4}(1-u^2)I(\|u\| \le 1)$	1
Biweight	2	$\frac{15}{16}(1-u^2)^2 I(\|u\| \le 1)$	1.0061
Triweight	3	$\frac{35}{32}(1-u^2)^3 I(\|u\| \le 1)$	1.0135
Gaussian	∞	$(\sqrt{2\pi})^{-1} \exp(-u^2/2)$	1.0513
Uniform	0	$\frac{1}{2}$	1.0758

Source: Fan and Gijbels (1996), p 44.

Table 3: Summary Statistics (for returns)

A. In-Sample Data

Series		Sample size	Mean	Standard Deviation	Skewness	Kurtosis
Short-term and Out-the-money	Call	1128	1.0606	6.5577	7.6792	66.7850
	Spot	1128	0.0002	0.0182	-0.1456	3.6117
Short-term and At-the-money	Call	1329	1.8142	14.2099	9.2917	99.4978
	Spot	1329	0.0001	0.0173	0.0904	3.1482
Short-term and In-the-money	Call	1388	1.0606	13.7979	15.3050	249.5262
	Spot	1388	0.0002	0.0162	0.6532	3.7502
Long-term and Out-the-money	Call	1078	0.8087	7.1224	10.2185	117.9250
	Spot	1078	0.0003	0.0183	-0.2391	5.6265
Long-term and At-the-money	Call	1246	1.3602	13.4516	10.4141	111.0757
	Spot	1246	0.0002	0.0171	0.2190	2.7178
Long-term and In-the-money	Call	1368	1.3759	17.9327	14.3542	214.4449
	Spot	1368	0.0002	0.0176	1.8063	19.5644

B. Out-of-Sample Data

Series		Sample size	Mean	Standard Deviation	Skewness	Kurtosis
Short-term and Out-the-money	Call	64	0.5819	4.3253	7.6793	60.1689
	Spot	64	-0.0010	0.0170	-2.0833	10.1946
Short-term and At-the-money	Call	109	0.7052	7.3233	10.3476	107.3728
	Spot	109	-0.0005	0.0148	-1.3524	5.4728
Short-term and In-the-money	Call	113	1.6884	17.8610	10.5789	111.9413
	Spot	113	-0.0005	0.0150	-0.9704	3.9165
Long-term and Out-the-money	Call	63	-0.0216	0.2928	0.9674	2.0542
	Spot	63	-0.0017	0.0180	-1.8276	8.0077
Long-term and At-the-money	Call	101	-0.0163	0.2483	0.8459	3.5016
	Spot	101	-0.0010	0.0152	-1.3243	5.1252
Long-term and In-the-money	Call	89	-0.0308	0.2387	0.9311	3.7580
	Spot	89	-0.0013	0.0161	-0.8188	3.6902

Note:

1. The short-term call option has a maturity of one month.
2. The long-term call option has a maturity of two months.
3. In-sample period begins from June 12, 1989 to December 31, 1999.
4. Out-of-sample period begins from January 1, 2000 to July 14, 2000.

Table 4: Bandwidths Used for Nonparametric Estimation

A. Epanechnikov Kernel

Series	Local Linear h_{CV}	Local Linear h_{PI}	Local Quadratic h_{CV}	Local Quadratic h_{PI}	Local Parametric h_{CV}	Local Parametric h_{PI}
Short-term and Out-the-money	0.04125	0.09063	0.04125	0.09063	0.00955	0.19150
Short-term and At-the-money	0.03025	0.06741	0.03025	0.06741	0.01385	0.15299
Short-term and In-the-money	0.02199	0.08331	0.02199	0.08331	0.02034	0.40060
Long-term and Out-the-money	0.06757	0.10182	0.06757	0.10182	0.01461	0.27029
Long-term and At-the-money	0.01959	0.06778	0.01959	0.06778	0.01411	0.25654
Long-term and In-the-money	0.10718	0.12462	0.10718	0.12462	0.03314	0.30295

B. Biweight Kernel

Series	Local Linear h_{CV}	Local Linear h_{PI}	Local Quadratic h_{CV}	Local Quadratic h_{PI}	Local Parametric h_{CV}	Local Parametric h_{PI}
Short-term and Out-the-money	0.04469	0.09285	0.04469	0.09285	0.01156	0.16440
Short-term and At-the-money	0.03025	0.06654	0.03025	0.06654	0.01524	0.13268
Short-term and In-the-money	0.02661	0.07166	0.02661	0.07166	0.03276	0.38517
Long-term and Out-the-money	0.06757	0.11923	0.06757	0.11923	0.01607	0.23480
Long-term and At-the-money	0.02371	0.07291	0.02371	0.07291	0.01707	0.22270
Long-term and In-the-money	0.10718	0.11431	0.10718	0.11431	0.03645	0.26730

Note:

1. h_{CV}'s are obtained from the minimization of the least squares cross-validation criterion (LSCV):

$$h_{CV} = \arg\min_h = n^{-1}\sum_{i=1}^{n}\{Y_i - \hat{m}_{h,-i}(X_i)\}^2 K(X_i).$$

2. h_{PI}'s are calculated from Gasser, Kneip and Köhler (1991):

$$h_{PI} = C_K\left(\frac{0.8\hat{\sigma}^2}{\int_{0.1}^{0.9}\left[\hat{m}_b''(x)\right]^2 dx}\right)^{1/5} n^{-1/5};$$

where $C_K = \left(J_K/\sigma_K^4\right)^{1/5}$, $J_K = \int_{-1}^{1} K^2(u)du$ and $\sigma_K^2 = \int_{-1}^{1} u^2 K(u)du$.

Table 5: In-Sample Evaluation of Hedging Performance for Short-term Options
(Estimated with Epanechnikov Kernel under Arbitrage Condition)

A. Short-term and Out-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	961.56	100	1323.60	939.69	699.10	100
N1 (CV)	2563.42	28.03	5223.62	1047.85	5159.92	60.66
N1 (PI)	1357.15	39.51	2111.06	1166.91	1773.83	55.74
N2 (CV)	5617.70	32.95	16360.99	884.69	16472.63	57.38
N2 (PI)	2356.39	37.87	4553.75	1467.30	4346.65	50.82
N3 (CV)	937.66	39.51	1162.79	779.84	869.67	57.38
N3 (PI)	994.59	32.95	1227.60	792.89	944.97	60.66
HALF1 (CV)	1651.59	31.31	2819.35	934.88	2681.91	60.66
HALF1 (PI)	1045.42	49.34	1450.52	994.03	1065.14	55.74
HALF2 (CV)	3137.96	34.59	8271.22	854.93	8295.20	57.38
HALF2 (PI)	1520.04	41.15	2534.03	1143.37	2280.18	50.82
HALF3 (CV)	872.74	39.51	1112.96	801.84	778.24	59.02
HALF3 (PI)	905.97	36.23	1145.46	808.31	818.34	60.66
SWITCH1 (CV)	2561.74	28.03	5226.08	1037.01	5164.67	60.66
SWITCH1 (PI)	1376.76	39.51	2136.11	1144.40	1818.65	54.10
SWITCH2 (CV)	5265.53	32.95	16094.11	507.34	16219.60	57.38
SWITCH2 (PI)	2393.76	37.87	4621.17	1460.33	4420.75	50.82
SWITCH3 (CV)	938.65	37.87	1162.55	778.85	870.24	57.38
SWITCH3 (PI)	990.79	31.31	1219.01	796.69	930.30	60.66

B. Short-term and At-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	537.40	100	862.45	488.17	492.78	100
N1 (CV)	3803.18	28.92	11012.54	1577.26	10973.41	58.11
N1 (PI)	1475.69	35.68	2651.73	849.81	2529.02	54.05
N2 (CV)	8086.36	26.22	24580.38	2846.56	24581.65	56.76
N2 (PI)	4512.76	20.81	8751.66	1519.41	8677.58	55.41
N3 (CV)	585.83	31.62	787.58	323.14	723.14	51.35
N3 (PI)	604.87	30.27	803.98	335.94	735.42	48.65
HALF1 (CV)	2063.62	31.62	5543.92	972.55	5495.20	58.11
HALF1 (PI)	902.12	41.08	1478.70	608.82	1356.74	54.05
HALF2 (CV)	4184.48	28.92	12324.78	1607.20	12302.94	56.76
HALF2 (PI)	2333.67	27.57	4400.51	943.63	4327.48	55.41
HALF3 (CV)	477.33	41.08	684.99	345.49	595.52	51.35
HALF3 (PI)	492.71	38.38	693.52	351.89	601.70	48.65
SWITCH1 (CV)	2760.09	30.27	7129.32	597.70	7152.71	55.41
SWITCH1 (PI)	1312.82	37.03	2281.23	782.61	2157.41	51.35
SWITCH2 (CV)	5803.57	27.57	16511.93	793.30	16605.44	55.41
SWITCH2 (PI)	3593.34	20.81	6672.44	1112.66	6623.92	54.05
SWITCH3 (CV)	559.38	32.97	771.74	342.09	696.49	45.95
SWITCH3 (PI)	599.04	32.97	799.71	346.71	725.56	48.65

Table 5 (Continued)

C. Short-term and In-the-money Series						
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	300.21	100	527.88	265.88	237.90	100
N1 (CV)	4583.78	12.33	7049.20	58.64	7090.30	56.98
N1 (PI)	417.88	30.93	602.76	158.37	584.99	50.00
N2 (CV)	6931.63	12.33	12142.45	-1629.40	12103.21	47.67
N2 (PI)	1392.96	27.44	2830.77	271.35	2834.26	45.35
N3 (CV)	395.74	26.28	529.82	140.32	513.89	51.16
N3 (PI)	502.66	23.95	661.27	155.86	646.41	48.84
HALF1 (CV)	2315.62	14.65	3545.53	99.89	3564.91	56.98
HALF1 (PI)	268.35	40.23	397.34	149.76	370.20	50.00
HALF2 (CV)	3470.42	14.65	6034.82	-744.13	6023.89	47.67
HALF2 (PI)	741.69	29.77	1446.85	206.25	1440.47	45.35
HALF3 (CV)	270.57	33.26	386.72	140.73	362.31	51.16
HALF3 (PI)	325.02	29.77	450.12	148.50	427.41	48.84
SWITCH1 (CV)	3104.12	15.81	5664.82	145.89	5696.15	47.67
SWITCH1 (PI)	283.25	46.05	432.49	170.25	399.90	50.00
SWITCH2 (CV)	4371.09	14.65	7606.97	-796.71	7609.50	45.35
SWITCH2 (PI)	757.29	30.93	1399.31	297.09	1375.43	55.81
SWITCH3 (CV)	267.43	37.91	417.64	122.29	401.67	41.86
SWITCH3 (PI)	326.00	32.09	491.08	120.55	478.85	41.86

Notes:

1. In-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under arbitrage condition (model N3), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The short-term call option has a maturity of one month.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Epanechnikov kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 6: In-Sample Evaluation of Hedging Performance for Long-term Options
(Estimated with Epanechnikov Kernel under Arbitrage Condition)

A. Long-term and Out-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	991.43	100	1428.79	918.07	882.89	100
N1 (CV)	1572.12	27.24	1994.23	1071.75	1711.52	65.52
N1 (PI)	1353.24	34.14	1634.87	1214.91	1113.34	68.97
N2 (CV)	3383.12	30.69	4988.97	1450.99	4857.80	62.07
N2 (PI)	2393.89	30.69	3014.65	1614.18	2591.16	62.07
N3 (CV)	1074.97	34.14	1384.30	748.24	1185.28	51.72
N3 (PI)	1238.77	20.34	1501.90	728.12	1336.86	62.07
HALF1 (CV)	1164.03	30.69	1466.81	917.42	1164.76	65.52
HALF1 (PI)	1049.87	37.59	1332.21	992.64	904.23	68.97
HALF2 (CV)	1949.96	34.14	2664.28	1109.86	2464.98	62.07
HALF2 (PI)	1520.81	34.14	1784.23	1176.77	1364.89	62.07
HALF3 (CV)	953.42	41.03	1268.38	765.72	1029.06	51.72
HALF3 (PI)	1035.83	23.79	1320.20	755.63	1101.73	58.62
SWITCH1 (CV)	1422.86	27.24	1770.61	1022.19	1471.34	55.17
SWITCH1 (PI)	1276.24	41.03	1603.92	1076.01	1210.49	62.07
SWITCH2 (CV)	2608.28	30.69	3782.82	1518.09	3526.17	55.17
SWITCH2 (PI)	2021.19	30.69	2721.51	1236.40	2467.36	55.17
SWITCH3 (CV)	1038.99	41.03	1361.76	769.85	1143.15	51.72
SWITCH3 (PI)	1224.48	23.79	1492.00	712.66	1333.99	58.62

B. Long-term and At-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	619.51	100	958.32	575.09	550.71	100
N1 (CV)	3988.26	24.29	6907.58	840.39	6956.36	51.43
N1 (PI)	1219.60	32.86	1685.25	934.00	1423.23	57.14
N2 (CV)	4733.76	12.86	6878.18	2621.52	6451.85	62.86
N2 (PI)	1564.04	38.57	2320.79	1252.93	1982.04	60.00
N3 (CV)	676.88	32.86	923.54	504.12	785.11	60.00
N3 (PI)	922.09	15.71	1134.47	406.92	1074.44	51.43
HALF1 (CV)	2094.39	38.57	3612.85	600.50	3614.61	51.43
HALF1 (PI)	801.30	41.43	1027.25	692.36	769.95	57.14
HALF2 (CV)	2562.87	21.43	3841.79	1333.47	3655.55	62.86
HALF2 (PI)	906.12	41.43	1272.04	852.40	957.98	60.00
HALF3 (CV)	558.98	41.43	798.60	469.42	655.50	57.14
HALF3 (PI)	672.13	32.86	894.58	422.21	800.19	51.43
SWITCH1 (CV)	2689.63	27.14	4682.95	244.36	4744.85	40.00
SWITCH1 (PI)	872.34	35.71	1101.24	553.07	966.18	54.29
SWITCH2 (CV)	3072.71	15.71	4385.80	716.30	4390.09	51.43
SWITCH2 (PI)	1251.19	38.57	1681.55	607.99	1590.67	54.29
SWITCH3 (CV)	664.53	30.00	907.07	437.12	806.40	51.43
SWITCH3 (PI)	871.20	15.71	1077.67	426.78	1004.00	51.43

Table 6 (Continued)

C. Long-term and In-the-money Series						
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	396.58	100	673.34	323.84	375.43	100
N1 (CV)	851.08	35.58	1289.36	263.16	1277.15	34.88
N1 (PI)	682.68	35.58	1006.16	265.88	981.88	39.53
N2 (CV)	1322.09	33.26	2618.75	8.69	2649.73	39.53
N2 (PI)	761.45	37.91	1113.37	330.38	1075.80	34.88
N3 (CV)	620.40	26.28	839.30	244.60	812.37	51.16
N3 (PI)	757.60	16.98	965.81	241.42	946.22	53.49
HALF1 (CV)	449.61	51.86	669.55	234.73	634.47	34.88
HALF1 (PI)	393.68	49.53	566.07	239.21	519.12	39.53
HALF2 (CV)	694.29	49.53	1377.87	110.40	1389.69	39.53
HALF2 (PI)	442.35	47.21	619.02	268.43	564.39	34.88
HALF3 (CV)	442.90	30.93	619.00	230.30	581.36	51.16
HALF3 (PI)	503.17	26.28	677.13	228.34	645.02	53.49
SWITCH1 (CV)	521.99	30.93	692.18	365.34	594.87	72.09
SWITCH1 (PI)	445.10	30.93	608.86	336.99	513.10	74.42
SWITCH2 (CV)	596.91	33.26	944.63	355.34	885.61	62.79
SWITCH2 (PI)	518.91	28.60	693.80	373.80	591.42	69.77
SWITCH3 (CV)	451.67	26.28	665.63	239.88	628.25	51.16
SWITCH3 (PI)	522.78	30.93	750.87	226.75	724.28	39.53

Notes:

1. In-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under arbitrage condition (model N3), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The long-term call option has a maturity of two months.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Epanechnikov kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 7: In-Sample Evaluation of Hedging Performance for Short-term Options
(Estimated with Epanechnikov Kernel under No Arbitrage Condition)

A. Short-term and Out-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	961.56	100	1323.60	939.69	699.10	100
N1 (CV)	2563.42	28.03	5223.62	1047.85	5159.92	60.66
N1 (PI)	1357.15	39.51	2111.06	1166.91	1773.83	55.74
N2 (CV)	5617.70	32.95	16360.99	884.69	16472.63	57.38
N2 (PI)	2356.39	37.87	4553.75	1467.30	4346.65	50.82
N4 (CV)	1003.19	29.67	1218.66	752.17	966.79	60.66
N4 (PI)	994.92	32.95	1228.03	792.56	945.82	60.66
HALF1 (CV)	1651.59	31.31	2819.35	934.88	2681.91	60.66
HALF1 (PI)	1045.42	49.34	1450.52	994.03	1065.14	55.74
HALF2 (CV)	3137.96	34.59	8271.22	854.93	8295.20	57.38
HALF2 (PI)	1520.04	41.15	2534.03	1143.37	2280.18	50.82
HALF4 (CV)	894.25	36.23	1135.26	787.44	824.55	60.66
HALF4 (PI)	905.90	36.23	1145.22	807.91	818.41	60.66
SWITCH1 (CV)	2561.74	28.03	5226.08	1037.01	5164.67	60.66
SWITCH1 (PI)	1376.76	39.51	2136.11	1144.40	1818.65	54.10
SWITCH2 (CV)	5265.53	32.95	16094.11	507.34	16219.60	57.38
SWITCH2 (PI)	2393.76	37.87	4621.17	1460.33	4420.75	50.82
SWITCH4 (CV)	991.66	29.67	1208.11	763.70	943.87	60.66
SWITCH4 (PI)	990.79	31.31	1219.01	796.69	930.30	60.66

B. Short-term and At-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	537.40	100	862.45	488.17	492.78	100
N1 (CV)	3803.18	28.92	11012.54	1577.26	10973.41	58.11
N1 (PI)	1475.69	35.68	2651.73	849.81	2529.02	54.05
N2 (CV)	8086.36	26.22	24580.38	2846.56	24581.65	56.76
N2 (PI)	4512.76	20.81	8751.66	1519.41	8677.58	55.41
N4 (CV)	700.70	18.11	899.94	339.77	839.03	58.11
N4 (PI)	702.47	19.46	895.42	322.93	840.86	55.41
HALF1 (CV)	2063.62	31.62	5543.92	972.55	5495.20	58.11
HALF1 (PI)	902.12	41.08	1478.70	608.82	1356.74	54.05
HALF2 (CV)	4184.48	28.92	12324.78	1607.20	12302.94	56.76
HALF2 (PI)	2333.67	27.57	4400.51	943.63	4327.48	55.41
HALF4 (CV)	535.32	28.92	738.11	353.81	652.21	58.11
HALF4 (PI)	540.91	27.57	736.23	345.39	654.62	55.41
SWITCH1 (CV)	2760.09	30.27	7129.32	597.70	7152.71	55.41
SWITCH1 (PI)	1312.82	37.03	2281.23	782.61	2157.41	51.35
SWITCH2 (CV)	5803.57	27.57	16511.93	793.30	16605.44	55.41
SWITCH2 (PI)	3593.34	20.81	6672.44	1112.66	6623.92	54.05
SWITCH4 (CV)	667.71	19.46	872.92	365.69	798.04	55.41
SWITCH4 (PI)	677.01	18.11	871.53	347.05	804.91	56.76

Table 7 (Continued)

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
C. Short-term and In-the-money Series						
BS	300.21	100	527.88	265.88	237.90	100
N1 (CV)	4583.78	12.33	7049.20	58.64	7090.30	56.98
N1 (PI)	417.88	30.93	602.76	158.37	584.99	50.00
N2 (CV)	6931.63	12.33	12142.45	-1629.40	12103.21	47.67
N2 (PI)	1392.96	27.44	2830.77	271.35	2834.26	45.35
N4 (CV)	489.34	25.12	635.63	160.70	618.59	47.67
N4 (PI)	502.67	23.95	661.27	155.86	646.41	48.84
HALF1 (CV)	2315.62	14.65	3545.53	99.89	3564.91	56.98
HALF1 (PI)	268.35	40.23	397.34	149.76	370.20	50.00
HALF2 (CV)	3470.42	14.65	6034.82	-744.13	6023.89	47.67
HALF2 (PI)	741.69	29.77	1446.85	206.25	1440.47	45.35
HALF4 (CV)	317.96	32.09	438.49	150.92	414.11	47.67
HALF4 (PI)	325.02	29.77	450.12	148.50	427.41	48.84
SWITCH1 (CV)	3104.12	15.81	5664.82	145.89	5696.15	47.67
SWITCH1 (PI)	283.25	46.05	432.49	170.25	399.90	50.00
SWITCH2 (CV)	4371.09	14.65	7606.97	-796.71	7609.50	45.35
SWITCH2 (PI)	757.29	30.93	1399.31	297.09	1375.43	55.81
SWITCH4 (CV)	320.82	32.09	484.73	125.39	470.98	41.86
SWITCH4 (PI)	326.00	32.09	491.08	120.55	478.85	41.86

Notes:

1. In-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under no arbitrage condition (model N4), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The short-term call option has a maturity of one month.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Epanechnikov kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 8: In-Sample Evaluation of Hedging Performance for Long-term Options
(Estimated with Epanechnikov Kernel under No Arbitrage Condition)

A. Long-term and Out-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	991.43	100	1428.79	918.07	882.89	100
N1 (CV)	1572.12	27.24	1994.23	1071.75	1711.52	65.52
N1 (PI)	1353.24	34.14	1634.87	1214.91	1113.34	68.97
N2 (CV)	3383.12	30.69	4988.97	1450.99	4857.80	62.07
N2 (PI)	2393.89	30.69	3014.65	1614.18	2591.16	62.07
N4 (CV)	1131.81	27.24	1421.00	719.35	1247.16	51.72
N4 (PI)	1228.47	20.34	1491.87	738.42	1319.25	62.07
HALF1 (CV)	1164.03	30.69	1466.81	917.42	1164.76	65.52
HALF1 (PI)	1049.87	37.59	1332.21	992.64	904.23	68.97
HALF2 (CV)	1949.96	34.14	2664.28	1109.86	2464.98	62.07
HALF2 (PI)	1520.81	34.14	1784.23	1176.77	1364.89	62.07
HALF4 (CV)	990.57	27.24	1286.88	756.58	1059.40	51.72
HALF4 (PI)	1031.81	23.79	1317.19	761.90	1093.49	58.62
SWITCH1 (CV)	1422.86	27.24	1770.61	1022.19	1471.34	55.17
SWITCH1 (PI)	1276.24	41.03	1603.92	1076.01	1210.49	62.07
SWITCH2 (CV)	2608.28	30.69	3782.82	1518.09	3526.17	55.17
SWITCH2 (PI)	2021.19	30.69	2721.51	1236.40	2467.36	55.17
SWITCH4 (CV)	1122.69	30.69	1421.02	698.72	1259.28	48.28
SWITCH4 (PI)	1224.48	23.79	1492.00	712.66	1333.99	58.62

B. Long-term and At-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	619.51	100	958.32	575.09	550.71	100
N1 (CV)	3988.26	24.29	6907.58	840.39	6956.36	51.43
N1 (PI)	1219.60	32.86	1685.25	934.00	1423.23	57.14
N2 (CV)	4733.76	12.86	6878.18	2621.52	6451.85	62.86
N2 (PI)	1564.04	38.57	2320.79	1252.93	1982.04	60.00
N4 (CV)	861.71	18.57	1065.82	420.71	993.56	54.29
N4 (PI)	916.87	15.71	1127.63	412.14	1064.94	51.43
HALF1 (CV)	2094.39	38.57	3612.85	600.50	3614.61	51.43
HALF1 (PI)	801.30	41.43	1027.25	692.36	769.95	57.14
HALF2 (CV)	2562.87	21.43	3841.79	1333.47	3655.55	62.86
HALF2 (PI)	906.12	41.43	1272.04	852.40	957.98	60.00
HALF4 (CV)	648.89	32.86	863.21	431.17	758.73	54.29
HALF4 (PI)	670.45	32.86	892.07	425.75	795.35	51.43
SWITCH1 (CV)	2689.63	27.14	4682.95	244.36	4744.85	40.00
SWITCH1 (PI)	872.34	35.71	1101.24	553.07	966.18	54.29
SWITCH2 (CV)	3072.71	15.71	4385.80	716.30	4390.09	51.43
SWITCH2 (PI)	1251.19	38.57	1681.55	607.99	1590.67	54.29
SWITCH4 (CV)	803.88	18.57	1015.22	403.88	945.02	48.57
SWITCH4 (PI)	871.20	15.71	1077.67	426.78	1004.00	51.43

Table 8 (Continued)

C. Long-term and In-the-money Series							
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	396.58	100	673.34	323.84	375.43	100
N1 (CV)	851.08	35.58	1289.36	263.16	1277.15	34.88
N1 (PI)	682.68	35.58	1006.16	265.88	981.88	39.53
N2 (CV)	1322.09	33.26	2618.75	8.69	2649.73	39.53
N2 (PI)	761.45	37.91	1113.37	330.38	1075.80	34.88
N4 (CV)	743.48	21.63	955.04	250.14	932.61	51.16
N4 (PI)	757.60	16.98	965.81	241.42	946.22	53.49
HALF1 (CV)	449.61	51.86	669.55	234.73	634.47	34.88
HALF1 (PI)	393.68	49.53	566.07	239.21	519.12	39.53
HALF2 (CV)	694.29	49.53	1377.87	110.40	1389.69	39.53
HALF2 (PI)	442.35	47.21	619.02	268.43	564.39	34.88
HALF4 (CV)	495.62	30.93	673.21	232.16	639.40	51.16
HALF4 (PI)	502.93	26.28	676.92	228.09	644.87	53.49
SWITCH1 (CV)	521.99	30.93	692.18	365.34	594.87	72.09
SWITCH1 (PI)	445.10	30.93	608.86	336.99	513.10	74.42
SWITCH2 (CV)	596.91	33.26	944.63	355.34	885.61	62.79
SWITCH2 (PI)	518.91	28.60	693.80	373.80	591.42	69.77
SWITCH4 (CV)	523.54	23.95	747.85	239.19	716.96	46.51
SWITCH4 (PI)	522.78	30.93	750.87	226.75	724.28	39.53

Notes:

1. In-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under no arbitrage condition (model N4), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The long-term call option has a maturity of two months.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Epanechnikov kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 9: In-Sample Evaluation of Hedging Performance for Short-term Options
(Estimated with Biweight Kernel under Arbitrage Condition)

A. Short-term and Out-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	961.56	100	1323.60	939.69	699.10	100
N1 (CV)	2708.04	31.31	5815.01	937.44	5786.57	60.66
N1 (PI)	1448.75	34.59	2246.91	1165.53	1936.92	55.74
N2 (CV)	5566.76	29.67	15840.87	903.08	15946.36	59.02
N2 (PI)	2627.87	37.87	5212.95	1435.87	5052.89	50.82
N3 (CV)	923.41	37.87	1153.16	778.89	857.41	57.38
N3 (PI)	994.49	32.95	1227.47	792.98	944.72	60.66
HALF1 (CV)	1723.60	34.59	3089.04	879.62	2985.73	60.66
HALF1 (PI)	1094.63	44.43	1510.53	993.68	1147.11	55.74
HALF2 (CV)	3112.04	31.31	8011.03	862.13	8030.60	59.02
HALF2 (PI)	1655.77	37.87	2842.85	1127.28	2631.46	50.82
HALF3 (CV)	866.18	39.51	1111.20	802.35	775.14	59.02
HALF3 (PI)	905.75	36.23	1145.14	808.19	818.02	60.66
SWITCH1 (CV)	2732.41	29.67	5821.47	901.65	5798.95	59.02
SWITCH1 (PI)	1464.48	34.59	2268.96	1163.99	1963.81	55.74
SWITCH2 (CV)	5208.02	29.67	15558.72	520.60	15679.05	59.02
SWITCH2 (PI)	2660.82	37.87	5262.13	1425.34	5107.45	50.82
SWITCH3 (CV)	925.25	37.87	1153.80	777.06	859.98	57.38
SWITCH3 (PI)	990.79	31.31	1219.01	796.69	930.30	60.66

B. Short-term and At-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	537.40	100	862.45	488.17	492.78	100
N1 (CV)	4277.26	24.86	12311.27	1649.42	12283.56	56.76
N1 (PI)	1644.65	35.68	3008.49	895.90	2891.60	54.05
N2 (CV)	8385.13	23.51	24743.56	3370.27	24680.29	60.81
N2 (PI)	5071.99	23.51	11041.18	1485.26	11015.51	56.76
N3 (CV)	597.40	27.57	793.65	321.67	730.50	51.35
N3 (PI)	604.77	30.27	802.53	332.73	735.29	48.65
HALF1 (CV)	2299.37	30.27	6188.07	1008.63	6146.99	56.76
HALF1 (PI)	985.68	38.38	1635.50	631.87	1518.81	54.05
HALF2 (CV)	4344.04	26.22	12409.59	1869.06	12351.77	60.81
HALF2 (PI)	2631.70	30.27	5533.51	926.55	5492.63	56.76
HALF3 (CV)	482.48	38.38	687.64	344.76	599.03	51.35
HALF3 (PI)	492.16	37.03	692.44	350.29	601.38	48.65
SWITCH1 (CV)	3084.01	24.86	8007.96	574.30	8041.86	54.05
SWITCH1 (PI)	1404.74	39.73	2434.32	757.28	2329.33	51.35
SWITCH2 (CV)	6001.74	26.22	16670.17	1227.49	16738.40	56.76
SWITCH2 (PI)	4059.14	23.51	8566.52	839.03	8583.52	55.41
SWITCH3 (CV)	564.91	30.27	775.78	343.01	700.58	50.00
SWITCH3 (PI)	598.28	32.97	797.24	344.14	724.05	48.65

Table 9 (Continued)

C. Short-term and In-the-money Series						
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	300.21	100	527.88	265.88	237.90	100
N1 (CV)	4434.92	14.65	7385.53	-34.07	7428.76	54.65
N1 (PI)	716.52	25.12	1216.93	182.42	1210.23	48.84
N2 (CV)	7081.28	13.49	12354.83	-1215.77	12366.98	47.67
N2 (PI)	1521.09	29.77	3089.31	457.66	3073.14	44.19
N3 (CV)	425.60	23.95	553.39	159.22	533.10	55.81
N3 (PI)	502.66	23.95	661.27	155.86	646.41	48.84
HALF1 (CV)	2244.97	16.98	3718.26	53.54	3739.68	54.65
HALF1 (PI)	419.86	26.28	680.57	161.78	664.94	48.84
HALF2 (CV)	3546.91	14.65	6145.16	-537.31	6157.53	47.67
HALF2 (PI)	796.05	34.42	1574.57	299.40	1554.91	44.19
HALF3 (CV)	284.49	29.77	398.29	150.18	371.05	55.81
HALF3 (PI)	325.02	29.77	450.12	148.50	427.41	48.84
SWITCH1 (CV)	2851.21	16.98	5373.45	294.25	5396.86	48.84
SWITCH1 (PI)	380.56	33.26	601.51	182.13	576.63	52.33
SWITCH2 (CV)	4266.52	15.81	7513.76	-805.64	7514.26	45.35
SWITCH2 (PI)	993.32	29.77	1840.83	396.25	1808.22	54.65
SWITCH3 (CV)	273.42	41.40	423.87	· 127.80	406.52	40.70
SWITCH3 (PI)	326.00	32.09	491.08	120.55	478.85	41.86

Notes:

1. In-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under arbitrage condition (model N3), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The short-term call option has a maturity of one month.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Biweight kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 10: In-Sample Evaluation of Hedging Performance for Long-term Options
(Estimated with Biweight Kernel under Arbitrage Condition)

A. Long-term and Out-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	991.43	100	1428.79	918.07	882.89	100
N1 (CV)	1660.42	23.79	2257.19	920.06	2097.65	68.97
N1 (PI)	1354.49	30.69	1645.47	1194.25	1152.00	65.52
N2 (CV)	3467.78	27.24	5436.26	931.57	5450.65	58.62
N2 (PI)	2401.27	30.69	2994.60	1628.28	2557.71	62.07
N3 (CV)	1067.62	34.14	1377.89	740.80	1182.37	55.17
N3 (PI)	1236.67	20.34	1500.33	730.22	1333.83	62.07
HALF1 (CV)	1209.78	34.14	1560.60	844.34	1335.69	68.97
HALF1 (PI)	1057.98	34.14	1336.22	980.91	923.41	65.52
HALF2 (CV)	1974.03	34.14	2899.10	859.29	2817.83	58.62
HALF2 (PI)	1528.55	34.14	1790.80	1187.82	1363.89	62.07
HALF3 (CV)	945.66	37.59	1264.82	755.96	1032.00	55.17
HALF3 (PI)	1035.60	23.79	1319.99	757.50	1100.13	58.62
SWITCH1 (CV)	1433.46	23.79	1751.28	972.06	1482.52	58.62
SWITCH1 (PI)	1287.33	37.59	1623.04	1079.49	1233.46	62.07
SWITCH2 (CV)	2594.09	27.24	3519.29	1311.38	3323.65	55.17
SWITCH2 (PI)	2037.20	34.14	2687.50	1249.69	2421.38	55.17
SWITCH3 (CV)	1038.11	37.59	1356.36	765.46	1139.54	55.17
SWITCH3 (PI)	1224.48	23.79	1492.00	712.66	1333.99	58.62

B. Long-term and At-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	619.51	100	958.32	575.09	550.71	100
N1 (CV)	3900.68	27.14	6809.73	785.39	6863.04	51.43
N1 (PI)	1351.86	38.57	1962.64	1060.44	1675.61	54.29
N2 (CV)	4546.78	15.71	6713.63	2351.48	6380.16	60.00
N2 (PI)	1686.73	35.71	2508.91	1291.46	2182.40	57.14
N3 (CV)	678.61	35.71	920.54	492.73	788.92	60.00
N3 (PI)	925.55	15.71	1139.33	403.46	1081.05	51.43
HALF1 (CV)	2056.05	38.57	3565.18	571.88	3570.39	51.43
HALF1 (PI)	850.09	47.14	1129.84	752.01	855.53	54.29
HALF2 (CV)	2432.87	30.00	3717.17	1242.06	3554.66	60.00
HALF2 (PI)	943.62	44.29	1342.79	873.93	1034.37	57.14
HALF3 (CV)	555.35	44.29	795.89	459.81	659.11	57.14
HALF3 (PI)	674.54	32.86	896.60	421.16	803.08	51.43
SWITCH1 (CV)	2528.54	30.00	4466.85	323.89	4520.13	40.00
SWITCH1 (PI)	912.84	32.86	1156.41	565.85	1023.24	54.29
SWITCH2 (CV)	2981.77	15.71	4344.90	632.32	4361.40	48.57
SWITCH2 (PI)	1388.64	30.00	1830.98	625.98	1745.77	54.29
SWITCH3 (CV)	667.69	32.86	905.35	433.85	806.22	48.57
SWITCH3 (PI)	871.20	15.71	1077.67	426.78	1004.00	51.43

Table 10 (Continued)

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
C. Long-term and In-the-money Series						
BS	396.58	100	673.34	323.84	375.43	100
N1 (CV)	1027.05	33.26	1525.52	220.51	1527.36	27.91
N1 (PI)	966.77	35.58	1447.89	234.55	1445.68	30.23
N2 (CV)	1489.67	33.26	2485.38	34.15	2514.56	39.53
N2 (PI)	1445.76	35.58	2573.40	12.76	2603.82	37.21
N3 (CV)	617.48	26.28	838.04	235.13	813.90	51.16
N3 (PI)	762.42	16.98	976.75	236.59	958.88	53.49
HALF1 (CV)	523.99	49.53	782.95	213.03	762.33	27.91
HALF1 (PI)	499.36	49.53	744.08	220.04	719.21	30.23
HALF2 (CV)	767.22	40.23	1292.67	125.95	1301.75	39.53
HALF2 (PI)	749.79	44.88	1343.59	113.47	1354.63	37.21
HALF3 (CV)	442.02	30.93	617.35	225.56	581.46	51.16
HALF3 (PI)	505.95	26.28	682.58	226.29	651.60	53.49
SWITCH1 (CV)	581.30	28.60	771.13	386.39	675.25	67.44
SWITCH1 (PI)	561.49	28.60	743.95	378.86	647.84	69.77
SWITCH2 (CV)	754.08	23.95	1108.17	309.89	1076.56	67.44
SWITCH2 (PI)	700.79	26.28	1045.09	327.96	1004.04	65.12
SWITCH3 (CV)	453.95	28.60	665.48	232.64	630.87	46.51
SWITCH3 (PI)	522.77	30.93	750.84	226.74	724.25	39.53

Notes:

1. In-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under arbitrage condition (model N3), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The long-term call option has a maturity of two months.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Biweight kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 11: In-Sample Evaluation of Hedging Performance for Short-term Options
(Estimated with Biweight Kernel under No Arbitrage Condition)

A. Short-term and Out-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	961.56	100	1323.60	939.69	699.10	100
N1 (CV)	2708.04	31.31	5815.01	937.44	5786.57	60.66
N1 (PI)	1448.75	34.59	2246.91	1165.53	1936.92	55.74
N2 (CV)	5566.76	29.67	15840.87	903.08	15946.36	59.02
N2 (PI)	2627.87	37.87	5212.95	1435.87	5052.89	50.82
N4 (CV)	1002.73	23.11	1215.62	753.17	962.10	62.30
N4 (PI)	994.58	32.95	1227.59	792.89	944.95	60.66
HALF1 (CV)	1723.60	34.59	3089.04	879.62	2985.73	60.66
HALF1 (PI)	1094.63	44.43	1510.53	993.68	1147.11	55.74
HALF2 (CV)	3112.04	31.31	8011.03	862.13	8030.60	59.02
HALF2 (PI)	1655.77	37.87	2842.85	1127.28	2631.46	50.82
HALF4 (CV)	893.73	31.31	1135.87	788.80	824.10	62.30
HALF4 (PI)	906.31	36.23	1146.02	808.66	818.79	60.66
SWITCH1 (CV)	2732.41	29.67	5821.47	901.65	5798.95	59.02
SWITCH1 (PI)	1464.48	34.59	2268.96	1163.99	1963.81	55.74
SWITCH2 (CV)	5208.02	29.67	15558.72	520.60	15679.05	59.02
SWITCH2 (PI)	2660.82	37.87	5262.13	1425.34	5107.45	50.82
SWITCH4 (CV)	992.32	23.11	1206.49	763.58	941.87	62.30
SWITCH4 (PI)	990.79	31.31	1219.01	796.69	930.30	60.66

B. Short-term and At-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	537.40	100	862.45	488.17	492.78	100
N1 (CV)	4277.26	24.86	12311.27	1649.42	12283.56	56.76
N1 (PI)	1644.65	35.68	3008.49	895.90	2891.60	54.05
N2 (CV)	8385.13	23.51	24743.56	3370.27	24680.29	60.81
N2 (PI)	5071.99	23.51	11041.18	1485.26	11015.51	56.76
N4 (CV)	703.97	18.11	902.35	337.03	842.76	58.11
N4 (PI)	701.77	19.46	896.43	321.34	842.57	55.41
HALF1 (CV)	2299.37	30.27	6188.07	1008.63	6146.99	56.76
HALF1 (PI)	985.68	38.38	1635.50	631.87	1518.81	54.05
HALF2 (CV)	4344.04	26.22	12409.59	1869.06	12351.77	60.81
HALF2 (PI)	2631.70	30.27	5533.51	926.55	5492.63	56.76
HALF4 (CV)	536.19	27.57	739.11	352.44	654.10	58.11
HALF4 (PI)	540.21	27.57	736.58	344.59	655.45	55.41
SWITCH1 (CV)	3084.01	24.86	8007.96	574.30	8041.86	54.05
SWITCH1 (PI)	1404.74	39.73	2434.32	757.28	2329.33	51.35
SWITCH2 (CV)	6001.74	26.22	16670.17	1227.49	16738.40	56.76
SWITCH2 (PI)	4059.14	23.51	8566.52	839.03	8583.52	55.41
SWITCH4 (CV)	664.42	19.46	872.88	365.79	797.95	55.41
SWITCH4 (PI)	675.96	18.11	871.97	345.84	805.92	55.41

Table 11 (Continued)

	C. Short-term and In-the-money Series					
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	300.21	100	527.88	265.88	237.90	100
N1 (CV)	4434.92	14.65	7385.53	-34.07	7428.76	54.65
N1 (PI)	716.52	25.12	1216.93	182.42	1210.23	48.84
N2 (CV)	7081.28	13.49	12354.83	-1215.77	12366.98	47.67
N2 (PI)	1521.09	29.77	3089.31	457.66	3073.14	44.19
N4 (CV)	498.74	25.12	644.38	155.81	628.92	48.84
N4 (PI)	502.66	23.95	661.27	155.86	646.41	48.84
HALF1 (CV)	2244.97	16.98	3718.26	53.54	3739.68	54.65
HALF1 (PI)	419.86	26.28	680.57	161.78	664.94	48.84
HALF2 (CV)	3546.91	14.65	6145.16	-537.31	6157.53	47.67
HALF2 (PI)	796.05	34.42	1574.57	299.40	1554.91	44.19
HALF4 (CV)	322.23	30.93	442.85	148.48	419.67	48.84
HALF4 (PI)	325.02	29.77	450.12	148.50	427.41	48.84
SWITCH1 (CV)	2851.21	16.98	5373.45	294.25	5396.86	48.84
SWITCH1 (PI)	380.56	33.26	601.51	182.13	576.63	52.33
SWITCH2 (CV)	4266.52	15.81	7513.76	-805.64	7514.26	45.35
SWITCH2 (PI)	993.32	29.77	1840.83	396.25	1808.22	54.65
SWITCH4 (CV)	323.47	32.09	486.52	122.17	473.69	41.86
SWITCH4 (PI)	326.00	32.09	491.08	120.55	478.85	41.86

Notes:

1. In-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under no arbitrage condition (model N4), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The short-term call option has a maturity of one month.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Biweight kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 12: In-Sample Evaluation of Hedging Performance for Long-term Options
(Estimated with Biweight Kernel under No Arbitrage Condition)

A. Long-term and Out-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	991.43	100	1428.79	918.07	882.89	100
N1 (CV)	1660.42	23.79	2257.19	920.06	2097.65	68.97
N1 (PI)	1354.49	30.69	1645.47	1194.25	1152.00	65.52
N2 (CV)	3467.78	27.24	5436.26	931.57	5450.65	58.62
N2 (PI)	2401.27	30.69	2994.60	1628.28	2557.71	62.07
N4 (CV)	1141.03	27.24	1423.76	726.83	1245.93	51.72
N4 (PI)	1226.37	20.34	1490.07	740.52	1315.92	62.07
HALF1 (CV)	1209.78	34.14	1560.60	844.34	1335.69	68.97
HALF1 (PI)	1057.98	34.14	1336.22	980.91	923.41	65.52
HALF2 (CV)	1974.03	34.14	2899.10	859.29	2817.83	58.62
HALF2 (PI)	1528.55	34.14	1790.80	1187.82	1363.89	62.07
HALF4 (CV)	991.07	30.69	1286.59	756.16	1059.36	48.28
HALF4 (PI)	1027.96	23.79	1315.73	760.15	1092.93	58.62
SWITCH1 (CV)	1433.46	23.79	1751.28	972.06	1482.52	58.62
SWITCH1 (PI)	1287.33	37.59	1623.04	1079.49	1233.46	62.07
SWITCH2 (CV)	2594.09	27.24	3519.29	1311.38	3323.65	55.17
SWITCH2 (PI)	2037.20	34.14	2687.50	1249.69	2421.38	55.17
SWITCH4 (CV)	1130.26	30.69	1422.86	707.85	1256.14	48.28
SWITCH4 (PI)	1224.48	23.79	1492.00	712.66	1333.99	58.62

B. Long-term and At-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	619.51	100	958.32	575.09	550.71	100
N1 (CV)	3900.68	27.14	6809.73	785.39	6863.04	51.43
N1 (PI)	1351.86	38.57	1962.64	1060.44	1675.61	54.29
N2 (CV)	4546.78	15.71	6713.63	2351.48	6380.16	60.00
N2 (PI)	1686.73	35.71	2508.91	1291.46	2182.40	57.14
N4 (CV)	865.83	21.43	1066.74	416.27	996.51	51.43
N4 (PI)	916.87	15.71	1127.63	412.14	1064.94	51.43
HALF1 (CV)	2056.05	38.57	3565.18	571.88	3570.39	51.43
HALF1 (PI)	850.09	47.14	1129.84	752.01	855.53	54.29
HALF2 (CV)	2432.87	30.00	3717.17	1242.06	3554.66	60.00
HALF2 (PI)	943.62	44.29	1342.79	873.93	1034.37	57.14
HALF4 (CV)	646.77	35.71	862.01	425.50	760.62	51.43
HALF4 (PI)	668.13	32.86	891.71	423.43	796.22	51.43
SWITCH1 (CV)	2528.54	30.00	4466.85	323.89	4520.13	40.00
SWITCH1 (PI)	912.84	32.86	1156.41	565.85	1023.24	54.29
SWITCH2 (CV)	2981.77	15.71	4344.90	632.32	4361.40	48.57
SWITCH2 (PI)	1388.64	30.00	1830.98	625.98	1745.77	54.29
SWITCH4 (CV)	809.42	21.43	1020.05	402.56	950.94	45.71
SWITCH4 (PI)	871.20	15.71	1077.67	426.78	1004.00	51.43

Table 12 (Continued)

C. Long-term and In-the-money Series						
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	396.58	100	673.34	323.84	375.43	100
N1 (CV)	1027.05	33.26	1525.52	220.51	1527.36	27.91
N1 (PI)	966.77	35.58	1447.89	234.55	1445.68	30.23
N2 (CV)	1489.67	33.26	2485.38	34.15	2514.56	39.53
N2 (PI)	1445.76	35.58	2573.40	12.76	2603.82	37.21
N4 (CV)	741.06	21.63	953.97	256.48	929.72	51.16
N4 (PI)	762.40	16.98	976.72	236.62	958.84	53.49
HALF1 (CV)	523.99	49.53	782.95	213.03	762.33	27.91
HALF1 (PI)	499.36	49.53	744.08	220.04	719.21	30.23
HALF2 (CV)	767.22	40.23	1292.67	125.95	1301.75	39.53
HALF2 (PI)	749.79	44.88	1343.59	113.47	1354.63	37.21
HALF4 (CV)	494.42	30.93	673.08	235.33	638.07	51.16
HALF4 (PI)	505.33	26.28	682.03	225.70	651.22	53.49
SWITCH1 (CV)	581.30	28.60	771.13	386.39	675.25	67.44
SWITCH1 (PI)	561.49	28.60	743.95	378.86	647.84	69.77
SWITCH2 (CV)	754.08	23.95	1108.17	309.89	1076.56	67.44
SWITCH2 (PI)	700.79	26.28	1045.09	327.96	1004.04	65.12
SWITCH4 (CV)	524.14	23.95	748.56	240.34	717.32	46.51
SWITCH4 (PI)	522.78	30.93	750.87	226.75	724.28	39.53

Notes:

1. In-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under no arbitrage condition (model N4), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The long-term call option has a maturity of two months.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Biweight kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 13: Out-of-Sample Evaluation of Hedging Performance for Short-term Options
(Estimated with Epanechnikov Kernel under Arbitrage Condition)

A. Short-term and Out-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	838.81	100	1116.49	838.81	540.96	100
N1 (CV)	819.16	50.00	986.40	808.19	632.27	60.00
N1 (PI)	843.88	30.00	937.72	843.88	457.15	80.00
N2 (CV)	900.26	70.00	1188.24	900.26	867.07	40.00
N2 (PI)	1085.42	50.00	1409.98	1041.51	1062.61	60.00
N3 (CV)	779.20	30.00	911.13	779.20	527.98	80.00
N3 (PI)	800.92	10.00	940.51	703.89	697.40	80.00
HALF1 (CV)	763.50	50.00	907.70	763.50	548.86	60.00
HALF1 (PI)	781.35	30.00	864.97	781.35	414.83	80.00
HALF2 (CV)	809.54	70.00	977.21	809.54	611.95	40.00
HALF2 (PI)	880.16	50.00	1017.42	880.16	570.59	60.00
HALF3 (CV)	749.01	30.00	888.22	749.01	533.76	80.00
HALF3 (PI)	734.81	30.00	899.66	711.35	615.80	80.00
SWITCH1 (CV)	819.16	50.00	986.40	808.19	632.27	60.00
SWITCH1 (PI)	843.88	30.00	937.72	843.88	457.15	80.00
SWITCH2 (CV)	900.26	70.00	1188.24	900.26	867.07	40.00
SWITCH2 (PI)	1085.42	50.00	1409.98	1041.51	1062.61	60.00
SWITCH3 (CV)	779.20	30.00	911.13	779.20	527.98	80.00
SWITCH3 (PI)	800.92	10.00	940.51	703.89	697.40	80.00

B. Short-term and At-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	470.63	100	785.76	415.90	482.43	100
N1 (CV)	1071.39	24.29	1275.49	649.77	1185.52	71.43
N1 (PI)	579.39	38.57	738.55	377.52	685.64	57.14
N2 (CV)	841.58	24.29	1067.45	568.22	976.04	71.43
N2 (PI)	1028.97	52.86	1280.13	355.92	1328.17	42.86
N3 (CV)	497.16	38.57	709.14	290.60	698.69	42.86
N3 (PI)	530.98	38.57	729.82	203.31	757.09	42.86
HALF1 (CV)	483.59	38.57	674.67	472.83	519.82	71.43
HALF1 (PI)	439.33	38.57	547.82	336.71	466.75	57.14
HALF2 (CV)	460.56	38.57	605.13	432.06	457.62	71.43
HALF2 (PI)	531.26	52.86	697.13	325.91	665.63	42.86
HALF3 (CV)	420.96	38.57	618.04	293.25	587.63	42.86
HALF3 (PI)	423.98	38.57	622.28	249.61	615.70	42.86
SWITCH1 (CV)	988.88	24.29	1253.42	545.85	1218.73	71.43
SWITCH1 (PI)	441.91	52.86	663.79	344.75	612.69	57.14
SWITCH2 (CV)	794.64	38.57	1057.16	434.98	1040.73	57.14
SWITCH2 (PI)	866.01	52.86	1157.88	335.26	1197.07	42.86
SWITCH3 (CV)	484.96	52.86	704.03	293.73	691.10	28.57
SWITCH3 (PI)	511.25	38.57	712.89	223.04	731.35	42.86

Table 13 (Continued)

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
C. Short-term and In-the-money Series						
BS	346.13	100	768.86	334.14	467.16	100
N1 (CV)	3126.02	10.00	4761.39	-61.53	4764.28	50.00
N1 (PI)	441.57	47.50	581.18	223.73	530.05	50.00
N2 (CV)	4714.98	10.00	6610.01	-1982.92	6263.59	37.50
N2 (PI)	918.12	10.00	1355.57	290.18	1323.34	62.50
N3 (CV)	464.66	22.50	804.19	125.73	799.26	25.00
N3 (PI)	456.60	35.00	831.26	140.05	818.04	25.00
HALF1 (CV)	1631.83	22.50	2590.51	76.31	2591.48	50.00
HALF1 (PI)	231.00	47.50	306.19	218.94	197.62	50.00
HALF2 (CV)	2429.74	10.00	3389.67	-884.39	3257.04	37.50
HALF2 (PI)	378.73	35.00	577.78	252.16	511.97	62.50
HALF3 (CV)	320.27	35.00	651.99	169.93	626.96	25.00
HALF3 (PI)	320.55	35.00	663.73	177.09	636.19	25.00
SWITCH1 (CV)	2721.46	10.00	4488.67	-62.95	4504.48	12.50
SWITCH1 (PI)	262.46	60.00	401.27	35.76	399.45	37.50
SWITCH2 (CV)	4234.07	10.00	6463.18	-1742.20	6194.00	25.00
SWITCH2 (PI)	889.78	22.50	1274.02	139.60	1265.28	50.00
SWITCH3 (CV)	351.01	35.00	777.77	184.31	753.67	25.00
SWITCH3 (PI)	381.86	35.00	803.85	158.62	787.58	25.00

Notes:

1. Out-of-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under arbitrage condition (model N3), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The short-term call option has a maturity of one month.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Epanechnikov kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 14: Out-of-Sample Evaluation of Hedging Performance for Long-term Options
(Estimated with Epanechnikov Kernel under Arbitrage Condition)

A. Long-term and Out-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	1378.44	100	1979.27	1378.44	1452.60	100
N1 (CV)	1893.77	10.00	2629.22	1667.06	2490.10	66.67
N1 (PI)	1566.62	43.33	2154.32	1406.44	1998.62	33.33
N2 (CV)	1694.91	76.67	2497.66	-1155.49	2711.96	0.00
N2 (PI)	931.79	76.67	1073.28	931.79	652.33	33.33
N3 (CV)	1392.98	10.00	1855.60	1272.05	1654.60	66.67
N3 (PI)	1585.20	10.00	2016.72	1265.29	1923.35	66.67
HALF1 (CV)	1566.01	10.00	2174.63	1462.75	1970.80	66.67
HALF1 (PI)	1402.44	43.33	1939.12	1332.44	1725.46	33.33
HALF2 (CV)	514.08	110.00	631.80	51.48	771.22	0.00
HALF2 (PI)	1095.12	76.67	1392.02	1095.12	1052.45	33.33
HALF3 (CV)	1315.62	10.00	1791.43	1265.25	1553.24	66.67
HALF3 (PI)	1411.73	10.00	1867.80	1261.87	1686.57	66.67
SWITCH1 (CV)	1893.77	10.00	2629.22	1667.06	2490.10	66.67
SWITCH1 (PI)	1566.62	43.33	2154.32	1406.44	1998.62	33.33
SWITCH2 (CV)	1694.91	76.67	2497.66	-1155.49	2711.96	0.00
SWITCH2 (PI)	931.79	76.67	1073.28	931.79	652.33	33.33
SWITCH3 (CV)	1392.98	10.00	1855.60	1272.05	1654.60	66.67
SWITCH3 (PI)	1585.20	10.00	2016.72	1265.29	1923.35	66.67

B. Long-term and At-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	846.15	100	1518.46	832.00	1212.18	100
N1 (CV)	2828.87	10.00	3656.73	-2056.12	3491.71	25.00
N1 (PI)	625.08	60.00	755.07	593.18	539.46	50.00
N2 (CV)	2885.99	10.00	3840.29	-1520.77	4071.87	25.00
N2 (PI)	654.60	60.00	931.01	225.23	1043.10	25.00
N3 (CV)	1064.98	10.00	1600.22	601.71	1712.17	50.00
N3 (PI)	1143.25	10.00	1682.93	609.50	1811.36	50.00
HALF1 (CV)	1051.36	35.00	1252.04	-672.06	1219.81	25.00
HALF1 (PI)	652.59	60.00	905.63	652.59	725.07	50.00
HALF2 (CV)	1079.92	35.00	1345.16	-404.38	1481.41	25.00
HALF2 (PI)	509.92	85.00	665.57	468.62	545.74	25.00
HALF3 (CV)	856.38	35.00	1423.76	656.85	1458.61	50.00
HALF3 (PI)	895.51	35.00	1463.00	660.75	1507.22	50.00
SWITCH1 (CV)	3332.49	10.00	3995.71	-2286.08	3784.10	25.00
SWITCH1 (PI)	533.80	60.00	619.10	509.49	406.13	50.00
SWITCH2 (CV)	3548.81	10.00	4168.30	-1857.19	4309.00	25.00
SWITCH2 (PI)	735.07	60.00	916.57	43.78	1057.15	25.00
SWITCH3 (CV)	1088.02	35.00	1603.00	564.37	1732.47	25.00
SWITCH3 (PI)	1222.04	10.00	1708.61	510.16	1882.93	50.00

Table 14 (Continued)

C. Long-term and In-the-money Series						
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	907.94	100	1762.11	907.94	1490.25	100
N1 (CV)	574.97	60.00	796.97	-378.37	809.94	25.00
N1 (PI)	272.18	35.00	318.28	-209.89	276.29	25.00
N2 (CV)	1000.22	35.00	1306.91	-685.17	1285.07	25.00
N2 (PI)	463.50	35.00	517.69	-315.32	474.10	25.00
N3 (CV)	1273.70	35.00	1999.14	693.85	2164.91	25.00
N3 (PI)	1365.39	10.00	2076.15	659.69	2273.09	25.00
HALF1 (CV)	284.33	60.00	402.46	204.78	400.06	25.00
HALF1 (PI)	367.19	60.00	628.31	289.02	644.19	25.00
HALF2 (CV)	249.71	35.00	255.38	51.38	288.86	25.00
HALF2 (PI)	379.28	35.00	555.19	236.31	580.11	25.00
HALF3 (CV)	1015.23	35.00	1744.98	740.89	1824.30	25.00
HALF3 (PI)	1044.46	35.00	1781.63	723.81	1879.83	25.00
SWITCH1 (CV)	406.74	85.00	748.86	-362.76	756.47	0.00
SWITCH1 (PI)	189.21	60.00	273.57	-170.32	247.21	0.00
SWITCH2 (CV)	666.97	35.00	1227.90	-569.75	1255.99	50.00
SWITCH2 (PI)	249.93	85.00	432.89	-222.76	428.59	0.00
SWITCH3 (CV)	1154.83	10.00	1968.88	835.93	2058.38	50.00
SWITCH3 (PI)	1232.59	35.00	2047.74	803.28	2175.00	25.00

Notes:

1. Out-of-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under arbitrage condition (model N3), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The long-term call option has a maturity of two months.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Epanechnikov kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 15: Out-of-Sample Evaluation of Hedging Performance for Short-term Options
(Estimated with Epanechnikov Kernel under No Arbitrage Condition)

A. Short-term and Out-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	838.81	100	1116.49	838.81	540.96	100
N1 (CV)	819.16	50.00	986.40	808.19	632.27	60.00
N1 (PI)	843.88	30.00	937.72	843.88	457.15	80.00
N2 (CV)	900.26	70.00	1188.24	900.26	867.07	40.00
N2 (PI)	1085.42	50.00	1409.98	1041.51	1062.61	60.00
N4 (CV)	750.98	30.00	924.98	750.98	603.76	80.00
N4 (PI)	800.92	10.00	940.51	703.89	697.40	80.00
HALF1 (CV)	763.50	50.00	907.70	763.50	548.86	60.00
HALF1 (PI)	781.35	30.00	864.97	781.35	414.83	80.00
HALF2 (CV)	809.54	70.00	977.21	809.54	611.95	40.00
HALF2 (PI)	880.16	50.00	1017.42	880.16	570.59	60.00
HALF4 (CV)	734.89	30.00	895.18	734.89	571.49	80.00
HALF4 (PI)	734.81	30.00	899.66	711.35	615.80	80.00
SWITCH1 (CV)	819.16	50.00	986.40	808.19	632.27	60.00
SWITCH1 (PI)	843.88	30.00	937.72	843.88	457.15	80.00
SWITCH2 (CV)	900.26	70.00	1188.24	900.26	867.07	40.00
SWITCH2 (PI)	1085.42	50.00	1409.98	1041.51	1062.61	60.00
SWITCH4 (CV)	750.98	30.00	924.98	750.98	603.76	80.00
SWITCH4 (PI)	800.92	10.00	940.51	703.89	697.40	80.00

B. Short-term and At-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	470.63	100	785.76	415.90	482.43	100
N1 (CV)	1071.39	24.29	1275.49	649.77	1185.52	71.43
N1 (PI)	579.39	38.57	738.55	377.52	685.64	57.14
N2 (CV)	841.58	24.29	1067.45	568.22	976.04	71.43
N2 (PI)	1028.97	52.86	1280.13	355.92	1328.17	42.86
N4 (CV)	574.83	10.00	748.18	234.51	767.40	57.14
N4 (PI)	597.01	24.29	755.37	203.28	785.79	42.86
HALF1 (CV)	483.59	38.57	674.67	472.83	519.82	71.43
HALF1 (PI)	439.33	38.57	547.82	336.71	466.75	57.14
HALF2 (CV)	460.56	38.57	605.13	432.06	457.62	71.43
HALF2 (PI)	531.26	52.86	697.13	325.91	665.63	42.86
HALF4 (CV)	454.16	24.29	629.78	265.20	616.99	57.14
HALF4 (PI)	465.25	24.29	631.55	249.59	626.63	42.86
SWITCH1 (CV)	988.88	24.29	1253.42	545.85	1218.73	71.43
SWITCH1 (PI)	441.91	52.86	663.79	344.75	612.69	57.14
SWITCH2 (CV)	794.64	38.57	1057.16	434.98	1040.73	57.14
SWITCH2 (PI)	866.01	52.86	1157.88	335.26	1197.07	42.86
SWITCH4 (CV)	572.27	10.00	746.99	224.42	769.57	57.14
SWITCH4 (PI)	596.05	24.29	757.07	191.59	791.11	42.86

Table 15 (Continued)

C. Short-term and In-the-money Series						
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	346.13	100	768.86	334.14	467.16	100
N1 (CV)	3126.02	10.00	4761.39	-61.53	4764.28	50.00
N1 (PI)	441.57	47.50	581.18	223.73	530.05	50.00
N2 (CV)	4714.98	10.00	6610.01	-1982.92	6263.59	37.50
N2 (PI)	918.12	10.00	1355.57	290.18	1323.34	62.50
N4 (CV)	449.62	35.00	829.16	133.58	817.15	25.00
N4 (PI)	456.61	35.00	831.26	140.05	818.04	25.00
HALF1 (CV)	1631.83	22.50	2590.51	76.31	2591.48	50.00
HALF1 (PI)	231.00	47.50	306.19	218.94	197.62	50.00
HALF2 (CV)	2429.74	10.00	3389.67	-884.39	3257.04	37.50
HALF2 (PI)	378.73	35.00	577.78	252.16	511.97	62.50
HALF4 (CV)	317.06	35.00	663.14	173.86	636.59	25.00
HALF4 (PI)	320.56	35.00	663.73	177.09	636.19	25.00
SWITCH1 (CV)	2721.46	10.00	4488.67	-62.95	4504.48	12.50
SWITCH1 (PI)	262.46	60.00	401.27	35.76	399.45	37.50
SWITCH2 (CV)	4234.07	10.00	6463.18	-1742.20	6194.00	25.00
SWITCH2 (PI)	889.78	22.50	1274.02	139.60	1265.28	50.00
SWITCH4 (CV)	384.66	22.50	803.93	152.16	789.12	12.50
SWITCH4 (PI)	381.87	35.00	803.85	158.63	787.58	25.00

Notes:

1. Out-of-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under no arbitrage condition (model N4), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The short-term call option has a maturity of one month.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Epanechnikov kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 16: Out-of-Sample Evaluation of Hedging Performance for Long-term Options
(Estimated with Epanechnikov Kernel under No Arbitrage Condition)

A. Long-term and Out-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	1378.44	100	1979.27	1378.44	1452.60	100
N1 (CV)	1893.77	10.00	2629.22	1667.06	2490.10	66.67
N1 (PI)	1566.62	43.33	2154.32	1406.44	1998.62	33.33
N2 (CV)	1694.91	76.67	2497.66	-1155.49	2711.96	0.00
N2 (PI)	931.79	76.67	1073.28	931.79	652.33	33.33
N4 (CV)	1403.92	10.00	1834.95	1230.09	1667.60	66.67
N4 (PI)	1585.20	10.00	2016.72	1265.29	1923.35	66.67
HALF1 (CV)	1566.01	10.00	2174.63	1462.75	1970.80	66.67
HALF1 (PI)	1402.44	43.33	1939.12	1332.44	1725.46	33.33
HALF2 (CV)	514.08	110.00	631.80	51.48	771.22	0.00
HALF2 (PI)	1095.12	76.67	1392.02	1095.12	1052.45	33.33
HALF4 (CV)	1321.09	10.00	1780.21	1244.27	1559.30	66.67
HALF4 (PI)	1411.73	10.00	1867.80	1261.87	1686.57	66.67
SWITCH1 (CV)	1893.77	10.00	2629.22	1667.06	2490.10	66.67
SWITCH1 (PI)	1566.62	43.33	2154.32	1406.44	1998.62	33.33
SWITCH2 (CV)	1694.91	76.67	2497.66	-1155.49	2711.96	0.00
SWITCH2 (PI)	931.79	76.67	1073.28	931.79	652.33	33.33
SWITCH4 (CV)	1403.92	10.00	1834.95	1230.09	1667.60	66.67
SWITCH4 (PI)	1585.20	10.00	2016.72	1265.29	1923.35	66.67

B. Long-term and At-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	846.15	100	1518.46	832.00	1212.18	100
N1 (CV)	2828.87	10.00	3656.73	-2056.12	3491.71	25.00
N1 (PI)	625.08	60.00	755.07	593.18	539.46	50.00
N2 (CV)	2885.99	10.00	3840.29	-1520.77	4071.87	25.00
N2 (PI)	654.60	60.00	931.01	225.23	1043.10	25.00
N4 (CV)	1201.84	10.00	1692.62	539.47	1852.54	50.00
N4 (PI)	1143.25	10.00	1682.93	609.50	1811.36	50.00
HALF1 (CV)	1051.36	35.00	1252.04	-672.06	1219.81	25.00
HALF1 (PI)	652.59	60.00	905.63	652.59	725.07	50.00
HALF2 (CV)	1079.92	35.00	1345.16	-404.38	1481.41	25.00
HALF2 (PI)	509.92	85.00	665.57	468.62	545.74	25.00
HALF4 (CV)	924.81	35.00	1459.60	625.74	1522.67	50.00
HALF4 (PI)	895.51	35.00	1463.00	660.75	1507.22	50.00
SWITCH1 (CV)	3332.49	10.00	3995.71	-2286.08	3784.10	25.00
SWITCH1 (PI)	533.80	60.00	619.10	509.49	406.13	50.00
SWITCH2 (CV)	3548.81	10.00	4168.30	-1857.19	4309.00	25.00
SWITCH2 (PI)	735.07	60.00	916.57	43.78	1057.15	25.00
SWITCH4 (CV)	1208.58	10.00	1675.91	512.19	1842.58	50.00
SWITCH4 (PI)	1222.04	10.00	1708.61	510.16	1882.93	50.00

Table 16 (Continued)

C. Long-term and In-the-money Series						
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	907.94	100	1762.11	907.94	1490.25	100
N1 (CV)	574.97	60.00	796.97	-378.37	809.94	25.00
N1 (PI)	272.18	35.00	318.28	-209.89	276.29	25.00
N2 (CV)	1000.22	35.00	1306.91	-685.17	1285.07	25.00
N2 (PI)	463.50	35.00	517.69	-315.32	474.10	25.00
N4 (CV)	1338.24	10.00	2035.02	647.30	2227.80	25.00
N4 (PI)	1365.39	10.00	2076.15	659.69	2273.09	25.00
HALF1 (CV)	284.33	60.00	402.46	204.78	400.06	25.00
HALF1 (PI)	367.19	60.00	628.31	289.02	644.19	25.00
HALF2 (CV)	249.71	35.00	255.38	51.38	288.86	25.00
HALF2 (PI)	379.28	35.00	555.19	236.31	580.11	25.00
HALF4 (CV)	1030.88	35.00	1761.27	717.62	1857.28	25.00
HALF4 (PI)	1044.46	35.00	1781.63	723.81	1879.83	25.00
SWITCH1 (CV)	406.74	85.00	748.86	-362.76	756.47	0.00
SWITCH1 (PI)	189.21	60.00	273.57	-170.32	247.21	0.00
SWITCH2 (CV)	666.97	35.00	1227.90	-569.75	1255.99	50.00
SWITCH2 (PI)	249.93	85.00	432.89	-222.76	428.59	0.00
SWITCH4 (CV)	1205.44	35.00	2007.10	790.89	2130.08	25.00
SWITCH4 (PI)	1232.59	35.00	2047.74	803.28	2175.00	25.00

Notes:

1. Out-of-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under no arbitrage condition (model N4), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The long-term call option has a maturity of two months.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Epanechnikov kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 17: Out-of-Sample Evaluation of Hedging Performance for Short-term Options
(Estimated with Biweight Kernel under Arbitrage Condition)

A. Short-term and Out-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	838.81	100	1116.49	838.81	540.96	100
N1 (CV)	814.57	50.00	977.32	811.97	608.13	60.00
N1 (PI)	836.81	30.00	951.01	836.81	505.18	80.00
N2 (CV)	888.54	70.00	1136.83	888.54	792.84	40.00
N2 (PI)	1000.43	50.00	1337.61	1000.43	992.68	60.00
N3 (CV)	777.34	30.00	909.85	777.34	528.64	80.00
N3 (PI)	800.92	10.00	940.51	703.89	697.40	80.00
HALF1 (CV)	765.39	50.00	904.73	765.39	539.35	60.00
HALF1 (PI)	777.81	30.00	879.39	777.81	458.73	80.00
HALF2 (CV)	803.68	70.00	960.32	803.68	587.71	40.00
HALF2 (PI)	859.62	50.00	1001.72	859.62	575.00	60.00
HALF3 (CV)	748.08	30.00	887.62	748.08	534.16	80.00
HALF3 (PI)	734.81	30.00	899.66	711.35	615.80	80.00
SWITCH1 (CV)	814.57	50.00	977.32	811.97	608.13	60.00
SWITCH1 (PI)	836.81	30.00	951.01	836.81	505.18	80.00
SWITCH2 (CV)	888.54	70.00	1136.83	888.54	792.84	40.00
SWITCH2 (PI)	1000.43	50.00	1337.61	1000.43	992.68	60.00
SWITCH3 (CV)	777.34	30.00	909.85	777.34	528.64	80.00
SWITCH3 (PI)	800.92	10.00	940.51	703.89	697.40	80.00

B. Short-term and At-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	470.63	100	785.76	415.90	482.43	100
N1 (CV)	1278.32	10.00	1525.50	597.04	1516.30	71.43
N1 (PI)	621.78	38.57	769.23	422.93	694.01	57.14
N2 (CV)	1344.54	10.00	1707.37	421.01	1787.23	71.43
N2 (PI)	608.86	38.57	725.50	296.60	715.16	57.14
N3 (CV)	487.26	24.29	701.28	306.41	681.34	57.14
N3 (PI)	529.72	38.57	728.84	210.81	753.59	42.86
HALF1 (CV)	582.24	38.57	733.19	446.47	628.17	71.43
HALF1 (PI)	460.53	38.57	572.44	359.42	481.25	57.14
HALF2 (CV)	615.35	38.57	770.54	358.45	736.73	71.43
HALF2 (PI)	418.61	52.86	479.01	296.25	406.57	57.14
HALF3 (CV)	416.92	24.29	615.70	301.16	580.05	57.14
HALF3 (PI)	424.90	38.57	622.16	253.35	613.77	42.86
SWITCH1 (CV)	1187.52	10.00	1494.21	478.23	1529.03	71.43
SWITCH1 (PI)	476.14	52.86	693.97	389.96	620.04	57.14
SWITCH2 (CV)	1250.44	24.29	1684.52	282.08	1793.80	71.43
SWITCH2 (PI)	582.10	24.29	695.96	212.31	715.89	42.86
SWITCH3 (CV)	490.12	38.57	704.62	296.43	690.45	42.86
SWITCH3 (PI)	509.99	38.57	711.79	230.54	727.38	42.86

Table 17 (Continued)

C. Short-term and In-the-money Series						
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	346.13	100	768.86	334.14	467.16	100
N1 (CV)	2942.48	10.00	4569.64	139.66	4570.94	37.50
N1 (PI)	366.86	35.00	504.54	221.10	446.91	50.00
N2 (CV)	4355.18	10.00	6299.86	-1946.40	5949.75	37.50
N2 (PI)	1003.59	22.50	1610.90	408.52	1556.70	50.00
N3 (CV)	463.11	22.50	814.77	116.29	808.89	25.00
N3 (PI)	456.60	35.00	831.26	140.05	818.04	25.00
HALF1 (CV)	1543.49	10.00	2500.58	176.90	2495.91	37.50
HALF1 (PI)	269.47	35.00	366.92	217.62	283.76	50.00
HALF2 (CV)	2249.84	10.00	3236.00	-866.13	3103.03	37.50
HALF2 (PI)	430.34	22.50	769.06	311.33	695.13	50.00
HALF3 (CV)	319.49	35.00	655.11	165.21	631.15	25.00
HALF3 (PI)	320.55	35.00	663.73	177.09	636.19	25.00
SWITCH1 (CV)	2455.12	22.50	4264.98	164.60	4276.79	25.00
SWITCH1 (PI)	242.71	60.00	346.36	98.18	332.05	25.00
SWITCH2 (CV)	3872.85	10.00	6176.18	-1721.14	5900.22	25.00
SWITCH2 (PI)	994.32	22.50	1477.66	333.98	1434.38	50.00
SWITCH3 (CV)	340.13	35.00	777.81	183.35	753.98	25.00
SWITCH3 (PI)	381.86	35.00	803.85	158.62	787.58	25.00

Notes:

1. Out-of-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under arbitrage condition (model N3), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The short-term call option has a maturity of one month.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Biweight kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 18: Out-of-Sample Evaluation of Hedging Performance for Long-term Options
(Estimated with Biweight Kernel under Arbitrage Condition)

A. Long-term and Out-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	1378.44	100	1979.27	1378.44	1452.60	100
N1 (CV)	1571.92	10.00	2040.43	1299.26	1926.89	66.67
N1 (PI)	1585.66	43.33	2162.78	1401.56	2017.38	33.33
N2 (CV)	1969.84	43.33	2884.91	-1392.94	3094.13	0.00
N2 (PI)	1142.80	76.67	1420.94	1142.80	1034.22	33.33
N3 (CV)	1365.03	10.00	1856.98	1290.42	1635.48	66.67
N3 (PI)	1585.20	10.00	2016.72	1265.29	1923.35	66.67
HALF1 (CV)	1405.09	10.00	1880.85	1278.85	1689.14	66.67
HALF1 (PI)	1411.96	43.33	1943.08	1330.00	1734.93	33.33
HALF2 (CV)	650.10	76.67	780.03	-67.25	951.77	0.00
HALF2 (PI)	1200.62	76.67	1571.80	1200.62	1242.41	33.33
HALF3 (CV)	1301.64	10.00	1792.61	1274.43	1544.00	66.67
HALF3 (PI)	1411.73	10.00	1867.80	1261.87	1686.57	66.67
SWITCH1 (CV)	1571.92	10.00	2040.43	1299.26	1926.89	66.67
SWITCH1 (PI)	1585.66	43.33	2162.78	1401.56	2017.38	33.33
SWITCH2 (CV)	1969.84	43.33	2884.91	-1392.94	3094.13	0.00
SWITCH2 (PI)	1142.80	76.67	1420.94	1142.80	1034.22	33.33
SWITCH3 (CV)	1365.03	10.00	1856.98	1290.42	1635.48	66.67
SWITCH3 (PI)	1585.20	10.00	2016.72	1265.29	1923.35	66.67

B. Long-term and At-the-money Series

Model	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	846.15	100	1518.46	832.00	1212.18	100
N1 (CV)	3169.65	10.00	4626.48	-2252.55	4666.23	25.00
N1 (PI)	599.82	60.00	801.61	552.67	670.46	50.00
N2 (CV)	2575.81	10.00	3460.66	-1104.01	3787.23	25.00
N2 (PI)	1029.99	60.00	1334.90	-60.13	1539.85	25.00
N3 (CV)	1100.77	10.00	1614.93	584.43	1738.36	50.00
N3 (PI)	1143.25	10.00	1682.93	609.50	1811.36	50.00
HALF1 (CV)	1221.75	35.00	1699.40	-770.28	1749.14	25.00
HALF1 (PI)	632.33	60.00	853.57	632.33	662.06	50.00
HALF2 (CV)	924.83	60.00	1183.56	-196.00	1347.78	25.00
HALF2 (PI)	370.10	85.00	514.03	325.94	458.97	25.00
HALF3 (CV)	874.27	35.00	1429.24	648.21	1470.85	50.00
HALF3 (PI)	895.51	35.00	1463.00	660.75	1507.22	50.00
SWITCH1 (CV)	3635.17	10.00	4825.88	-2531.49	4744.21	25.00
SWITCH1 (PI)	491.33	60.00	612.19	450.02	479.25	50.00
SWITCH2 (CV)	3204.49	10.00	3765.36	-1429.26	4022.47	25.00
SWITCH2 (PI)	1158.05	60.00	1357.17	-264.57	1537.06	25.00
SWITCH3 (CV)	1126.21	10.00	1621.09	544.68	1763.04	50.00
SWITCH3 (PI)	1222.04	10.00	1708.61	510.16	1882.93	50.00

Table 18 (Continued)

C. Long-term and In-the-money Series						
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	907.94	100	1762.11	907.94	1490.25	100
N1 (CV)	828.31	60.00	1193.55	-526.94	1236.61	25.00
N1 (PI)	757.93	60.00	1082.84	-485.67	1117.54	25.00
N2 (CV)	1488.38	10.00	2015.12	-1001.04	2019.45	25.00
N2 (PI)	1342.58	10.00	1802.89	-905.07	1800.47	25.00
N3 (CV)	1262.88	35.00	1995.65	704.66	2155.94	25.00
N3 (PI)	1365.32	10.00	2076.01	659.61	2272.95	25.00
HALF1 (CV)	224.30	60.00	260.11	130.50	259.81	25.00
HALF1 (PI)	240.85	60.00	292.94	151.13	289.77	25.00
HALF2 (CV)	351.02	35.00	365.77	-106.55	404.04	25.00
HALF2 (PI)	278.13	35.00	294.64	-58.57	333.44	25.00
HALF3 (CV)	1009.82	35.00	1743.99	746.30	1820.09	25.00
HALF3 (PI)	1044.42	35.00	1781.56	723.78	1879.75	25.00
SWITCH1 (CV)	617.05	60.00	1135.67	-516.52	1167.88	50.00
SWITCH1 (PI)	552.47	60.00	1027.84	-473.77	1053.25	50.00
SWITCH2 (CV)	1052.43	10.00	1915.21	-858.84	1976.67	75.00
SWITCH2 (PI)	934.52	10.00	1709.35	-771.53	1761.29	75.00
SWITCH3 (CV)	1154.83	10.00	1968.88	835.93	2058.38	50.00
SWITCH3 (PI)	1232.52	35.00	2047.59	803.21	2174.85	25.00

Notes:

1. Out-of-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under arbitrage condition (model N3), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The long-term call option has a maturity of two months.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Biweight kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 19: Out-of-Sample Evaluation of Hedging Performance for Short-term Options
(Estimated with Biweight Kernel under No Arbitrage Condition)

A. Short-term and Out-the-money Series

Model	Absolute Tracking Error Mean	%[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	S.D.[c]	%[b]
BS	838.81	100	1116.49	838.81	540.96	100
N1 (CV)	814.57	50.00	977.32	811.97	608.13	60.00
N1 (PI)	836.81	30.00	951.01	836.81	505.18	80.00
N2 (CV)	888.54	70.00	1136.83	888.54	792.84	40.00
N2 (PI)	1000.43	50.00	1337.61	1000.43	992.68	60.00
N4 (CV)	748.42	30.00	923.75	748.42	605.38	80.00
N4 (PI)	800.92	10.00	940.51	703.89	697.40	80.00
HALF1 (CV)	765.39	50.00	904.73	765.39	539.35	60.00
HALF1 (PI)	777.81	30.00	879.39	777.81	458.73	80.00
HALF2 (CV)	803.68	70.00	960.32	803.68	587.71	40.00
HALF2 (PI)	859.62	50.00	1001.72	859.62	575.00	60.00
HALF4 (CV)	733.61	30.00	894.51	733.61	572.24	80.00
HALF4 (PI)	734.81	30.00	899.66	711.35	615.80	80.00
SWITCH1 (CV)	814.57	50.00	977.32	811.97	608.13	60.00
SWITCH1 (PI)	836.81	30.00	951.01	836.81	505.18	80.00
SWITCH2 (CV)	888.54	70.00	1136.83	888.54	792.84	40.00
SWITCH2 (PI)	1000.43	50.00	1337.61	1000.43	992.68	60.00
SWITCH4 (CV)	748.42	30.00	923.75	748.42	605.38	80.00
SWITCH4 (PI)	800.92	10.00	940.51	703.89	697.40	80.00

B. Short-term and At-the-money Series

Model	Absolute Tracking Error Mean	%[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	S.D.[c]	%[b]
BS	470.63	100	785.76	415.90	482.43	100
N1 (CV)	1278.32	10.00	1525.50	597.04	1516.30	71.43
N1 (PI)	621.78	38.57	769.23	422.93	694.01	57.14
N2 (CV)	1344.54	10.00	1707.37	421.01	1787.23	71.43
N2 (PI)	608.86	38.57	725.50	296.60	715.16	57.14
N4 (CV)	579.59	10.00	751.45	232.59	771.80	57.14
N4 (PI)	586.11	24.29	752.28	206.66	781.30	42.86
HALF1 (CV)	582.24	38.57	733.19	446.47	628.17	71.43
HALF1 (PI)	460.53	38.57	572.44	359.42	481.25	57.14
HALF2 (CV)	615.35	38.57	770.54	358.45	736.73	71.43
HALF2 (PI)	418.61	52.86	479.01	296.25	406.57	57.14
HALF4 (CV)	456.54	10.00	630.75	264.24	618.62	57.14
HALF4 (PI)	459.80	24.29	629.72	251.28	623.68	42.86
SWITCH1 (CV)	1187.52	10.00	1494.21	478.23	1529.03	71.43
SWITCH1 (PI)	476.14	52.86	693.97	389.96	620.04	57.14
SWITCH2 (CV)	1250.44	24.29	1684.52	282.08	1793.80	71.43
SWITCH2 (PI)	582.10	24.29	695.96	212.31	715.89	42.86
SWITCH4 (CV)	574.02	10.00	746.59	225.51	768.74	57.14
SWITCH4 (PI)	588.71	24.29	753.77	191.41	787.48	42.86

Table 19 (Continued)

C. Short-term and In-the-money Series						
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	346.13	100	768.86	334.14	467.16	100
N1 (CV)	2942.48	10.00	4569.64	139.66	4570.94	37.50
N1 (PI)	366.86	35.00	504.54	221.10	446.91	50.00
N2 (CV)	4355.18	10.00	6299.86	-1946.40	5949.75	37.50
N2 (PI)	1003.59	22.50	1610.90	408.52	1556.70	50.00
N4 (CV)	457.60	35.00	831.35	141.37	817.87	25.00
N4 (PI)	456.60	35.00	831.26	140.05	818.04	25.00
HALF1 (CV)	1543.49	10.00	2500.58	176.90	2495.91	37.50
HALF1 (PI)	269.47	35.00	366.92	217.62	283.76	50.00
HALF2 (CV)	2249.84	10.00	3236.00	-866.13	3103.03	37.50
HALF2 (PI)	430.34	22.50	769.06	311.33	695.13	50.00
HALF4 (CV)	321.05	35.00	663.78	177.75	636.04	25.00
HALF4 (PI)	320.55	35.00	663.73	177.09	636.19	25.00
SWITCH1 (CV)	2455.12	22.50	4264.98	164.60	4276.79	25.00
SWITCH1 (PI)	242.71	60.00	346.36	98.18	332.05	25.00
SWITCH2 (CV)	3872.85	10.00	6176.18	-1721.14	5900.22	25.00
SWITCH2 (PI)	994.32	22.50	1477.66	333.98	1434.38	50.00
SWITCH4 (CV)	382.99	22.50	803.88	159.82	787.34	25.00
SWITCH4 (PI)	381.86	35.00	803.85	158.62	787.58	25.00

Notes:

1. Out-of-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under no arbitrage condition (model N4), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The short-term call option has a maturity of one month.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Biweight kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Table 20: Out-of-Sample Evaluation of Hedging Performance for Long-term Options
(Estimated with Biweight Kernel under No Arbitrage Condition)

A. Long-term and Out-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	1378.44	100	1979.27	1378.44	1452.60	100
N1 (CV)	1571.92	10.00	2040.43	1299.26	1926.89	66.67
N1 (PI)	1585.66	43.33	2162.78	1401.56	2017.38	33.33
N2 (CV)	1969.84	43.33	2884.91	-1392.94	3094.13	0.00
N2 (PI)	1142.80	76.67	1420.94	1142.80	1034.22	33.33
N4 (CV)	1401.46	10.00	1844.69	1245.20	1666.89	66.67
N4 (PI)	1585.20	10.00	2016.72	1265.29	1923.35	66.67
HALF1 (CV)	1405.09	10.00	1880.85	1278.85	1689.14	66.67
HALF1 (PI)	1411.96	43.33	1943.08	1330.00	1734.93	33.33
HALF2 (CV)	650.10	76.67	780.03	-67.25	951.77	0.00
HALF2 (PI)	1200.62	76.67	1571.80	1200.62	1242.41	33.33
HALF4 (CV)	1319.86	10.00	1785.41	1251.82	1559.15	66.67
HALF4 (PI)	1411.73	10.00	1867.80	1261.87	1686.57	66.67
SWITCH1 (CV)	1571.92	10.00	2040.43	1299.26	1926.89	66.67
SWITCH1 (PI)	1585.66	43.33	2162.78	1401.56	2017.38	33.33
SWITCH2 (CV)	1969.84	43.33	2884.91	-1392.94	3094.13	0.00
SWITCH2 (PI)	1142.80	76.67	1420.94	1142.80	1034.22	33.33
SWITCH4 (CV)	1401.46	10.00	1844.69	1245.20	1666.89	66.67
SWITCH4 (PI)	1585.20	10.00	2016.72	1265.29	1923.35	66.67

B. Long-term and At-the-money Series

Model	Absolute Tracking Error Mean	Absolute Tracking Error %[b]	Squared Tracking Error Root Mean	Tracking Error[a] Mean	Tracking Error[a] S.D.[c]	Tracking Error[a] %[b]
BS	846.15	100	1518.46	832.00	1212.18	100
N1 (CV)	3169.65	10.00	4626.48	-2252.55	4666.23	25.00
N1 (PI)	599.82	60.00	801.61	552.67	670.46	50.00
N2 (CV)	2575.81	10.00	3460.66	-1104.01	3787.23	25.00
N2 (PI)	1029.99	60.00	1334.90	-60.13	1539.85	25.00
N4 (CV)	1211.49	10.00	1693.94	532.16	1856.97	50.00
N4 (PI)	1143.25	10.00	1682.93	609.50	1811.36	50.00
HALF1 (CV)	1221.75	35.00	1699.40	-770.28	1749.14	25.00
HALF1 (PI)	632.33	60.00	853.57	632.33	662.06	50.00
HALF2 (CV)	924.83	60.00	1183.56	-196.00	1347.78	25.00
HALF2 (PI)	370.10	85.00	514.03	325.94	458.97	25.00
HALF4 (CV)	929.63	35.00	1459.68	622.08	1524.77	50.00
HALF4 (PI)	895.51	35.00	1463.00	660.75	1507.22	50.00
SWITCH1 (CV)	3635.17	10.00	4825.88	-2531.49	4744.21	25.00
SWITCH1 (PI)	491.33	60.00	612.19	450.02	479.25	50.00
SWITCH2 (CV)	3204.49	10.00	3765.36	-1429.26	4022.47	25.00
SWITCH2 (PI)	1158.05	60.00	1357.17	-264.57	1537.06	25.00
SWITCH4 (CV)	1217.54	10.00	1678.19	505.55	1847.78	50.00
SWITCH4 (PI)	1222.04	10.00	1708.61	510.16	1882.93	50.00

Table 20 (Continued)

C. Long-term and In-the-money Series						
	Absolute Tracking Error		Squared Tracking Error	Tracking Error[a]		
Model	Mean	%[b]	Root Mean	Mean	S.D.[c]	%[b]
BS	907.94	100	1762.11	907.94	1490.25	100
N1 (CV)	828.31	60.00	1193.55	-526.94	1236.61	25.00
N1 (PI)	757.93	60.00	1082.84	-485.67	1117.54	25.00
N2 (CV)	1488.38	10.00	2015.12	-1001.04	2019.45	25.00
N2 (PI)	1342.58	10.00	1802.89	-905.07	1800.47	25.00
N4 (CV)	1345.17	10.00	2037.48	640.37	2233.46	25.00
N4 (PI)	1365.39	10.00	2076.15	659.69	2273.09	25.00
HALF1 (CV)	224.30	60.00	260.11	130.50	259.81	25.00
HALF1 (PI)	240.85	60.00	292.94	151.13	289.77	25.00
HALF2 (CV)	351.02	35.00	365.77	-106.55	404.04	25.00
HALF2 (PI)	278.13	35.00	294.64	-58.57	333.44	25.00
HALF4 (CV)	1034.35	35.00	1761.98	714.15	1859.95	25.00
HALF4 (PI)	1044.46	35.00	1781.63	723.81	1879.83	25.00
SWITCH1 (CV)	617.05	60.00	1135.67	-516.52	1167.88	50.00
SWITCH1 (PI)	552.47	60.00	1027.84	-473.77	1053.25	50.00
SWITCH2 (CV)	1052.43	10.00	1915.21	-858.84	1976.67	75.00
SWITCH2 (PI)	934.52	10.00	1709.35	-771.53	1761.29	75.00
SWITCH4 (CV)	1212.37	35.00	2008.54	783.96	2135.31	25.00
SWITCH4 (PI)	1232.59	35.00	2047.74	803.28	2175.00	25.00

Notes:

1. Out-of-sample hedge portfolio weights are determined by (i) Black and Scholes model (model BS), (ii) local linear estimation (model N1), (iii) local quadratic estimation (model N2), (iv) local parametric estimation under no arbitrage condition (model N4), (v) weights that are an equally weighted average of those under BS model and each nonparametric model respectively (model HALF), and (vi) weights that switch from those under each nonparametric model respectively to those under BS model when the moneyness is above 1.05 (model SWITCH).
2. The long-term call option has a maturity of two months.
3. Least squares cross-validation bandwidth (CV) and Plug-in rule bandwidth (PI) with Biweight kernel function are used to estimate the hedge portfolio weights under each nonparametric model.

[a] The tracking error is defined as the maturity-date dollar payoff of a portfolio that long in one call and short in the hedge portfolio and is calculated by using the actual call price. [b] '%' denotes the frequency that the indicated procedure outperform BS. [c] 'S.D.' denotes standard deviation.

Figure 1 Deltas from Black and Scholes Model (BS) for Short-term and Out-the-money Series (In-Sample)

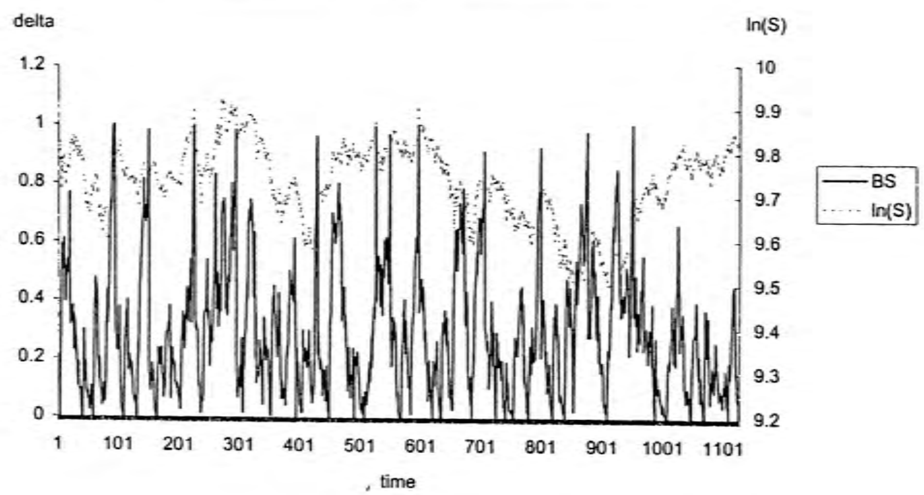

Figure 2 Deltas from Black and Scholes Model (BS) for Short-term and At-the-money Series (In-Sample)

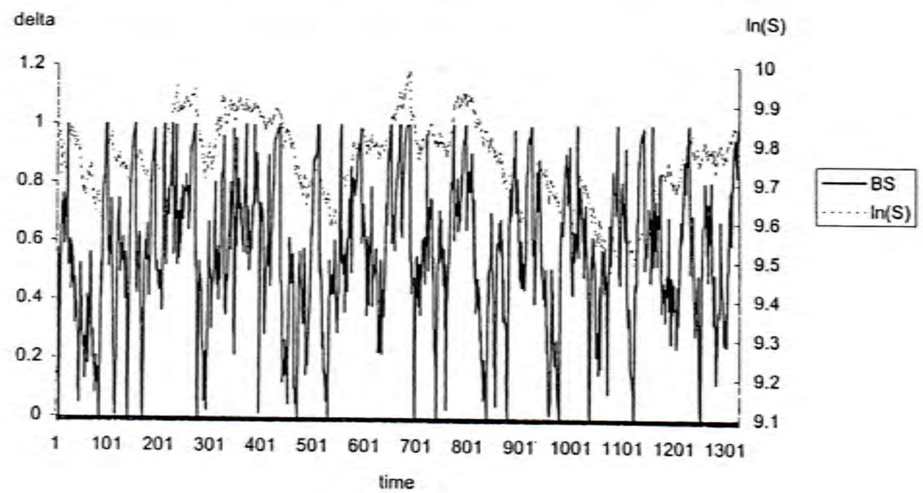

Figure 3 Deltas from Black and Scholes Model (BS) for Short-term and In-the-money Series (In-Sample)

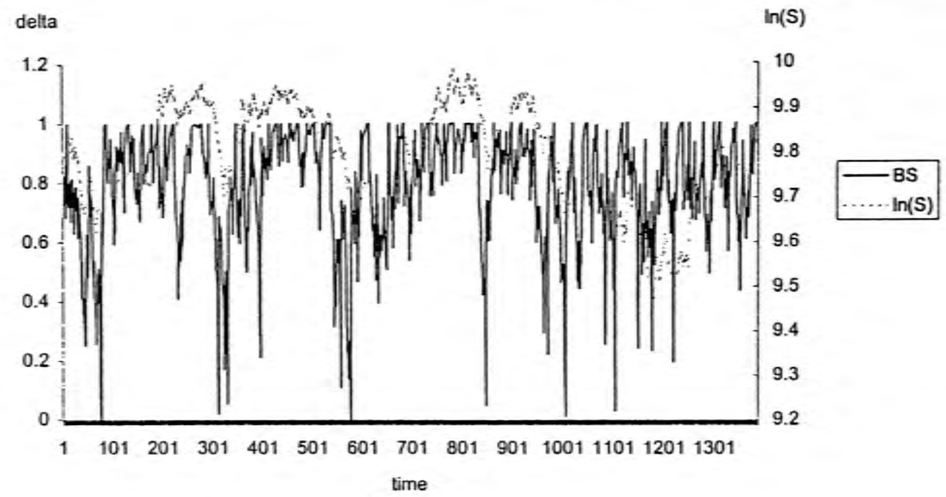

Figure 4 Deltas from Black and Scholes Model (BS) for Long-term and Out-the-money Series (In-Sample)

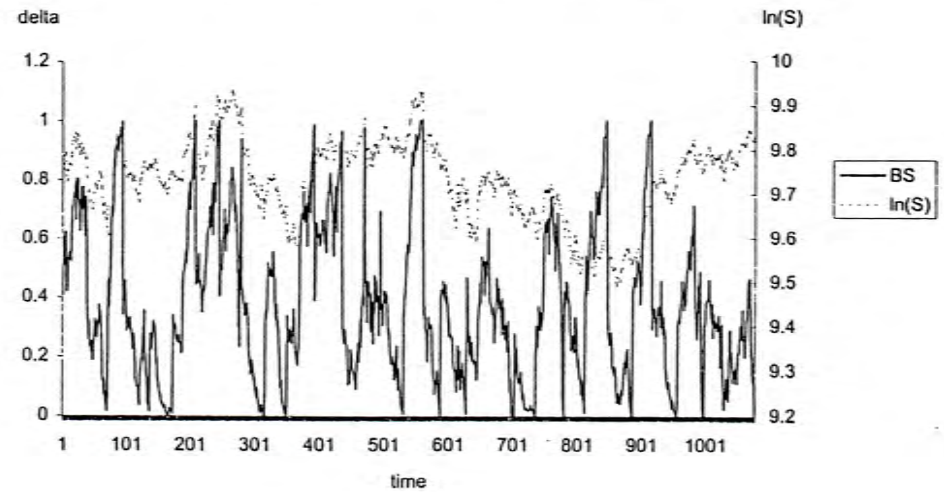

Figure 5 Deltas from Black and Scholes Model (BS) for Long-term and At-the-money Series (In-Sample)

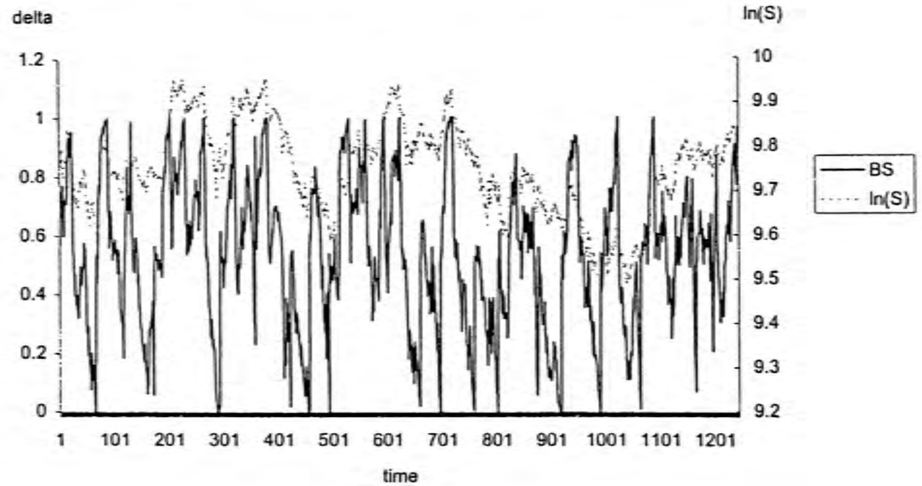

Figure 6 Deltas from Black and Scholes Model (BS) for Long-term and In-the-money Series (In-Sample)

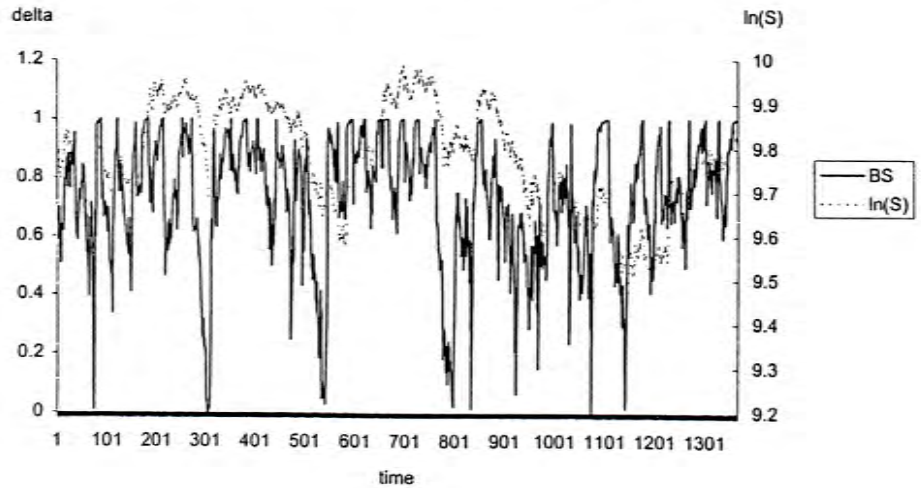

Figure 7 The Variation of Deltas from Black and Scholes Model (BS) with the Time-to-maturity for Short-term Options (June 2000)

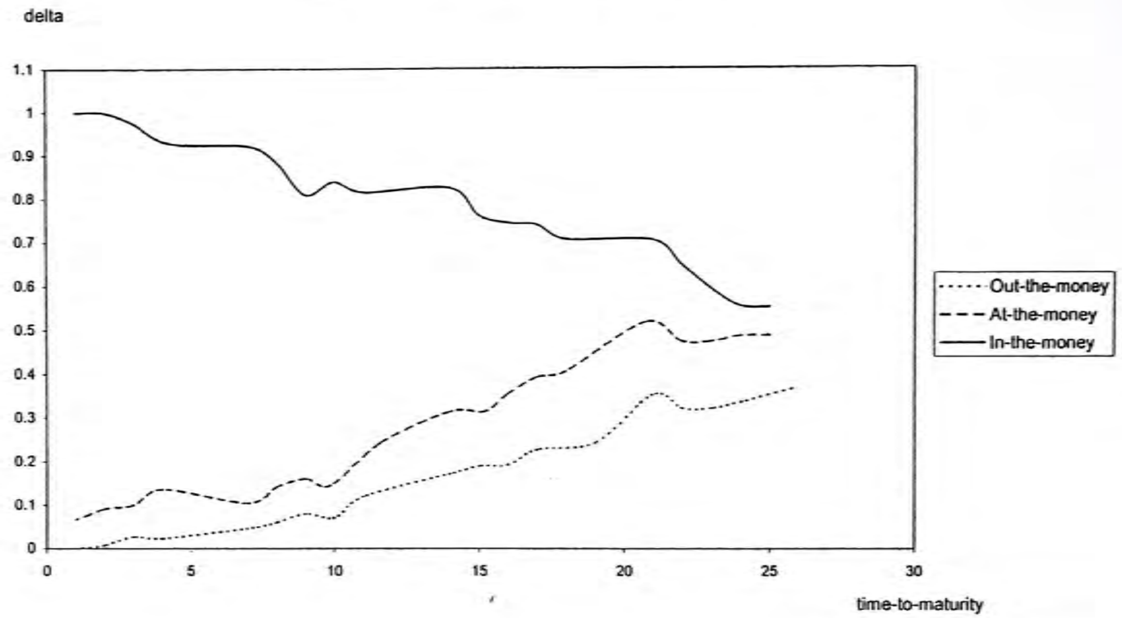

Figure 8 The Variation of Deltas from Black and Scholes Model (BS) with the Time-to-maturity for Long-term Options (June 2000)

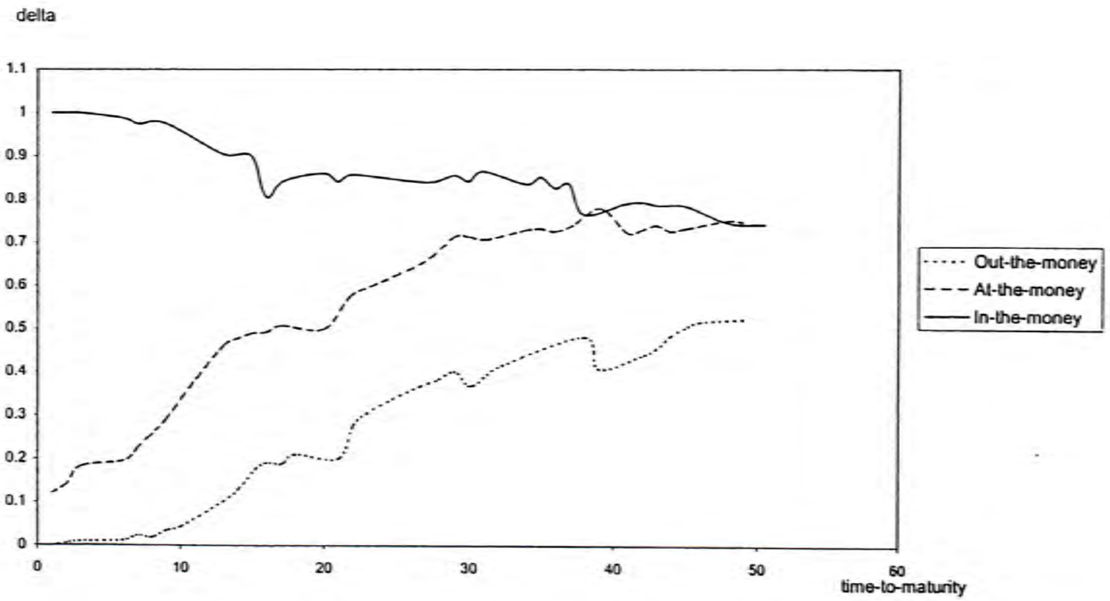

Figure 9 Deltas from Nonparametric Model (N1 with Cross-Validation Bandwidth)
for Short-term and Out-the-money Series (In-Sample)

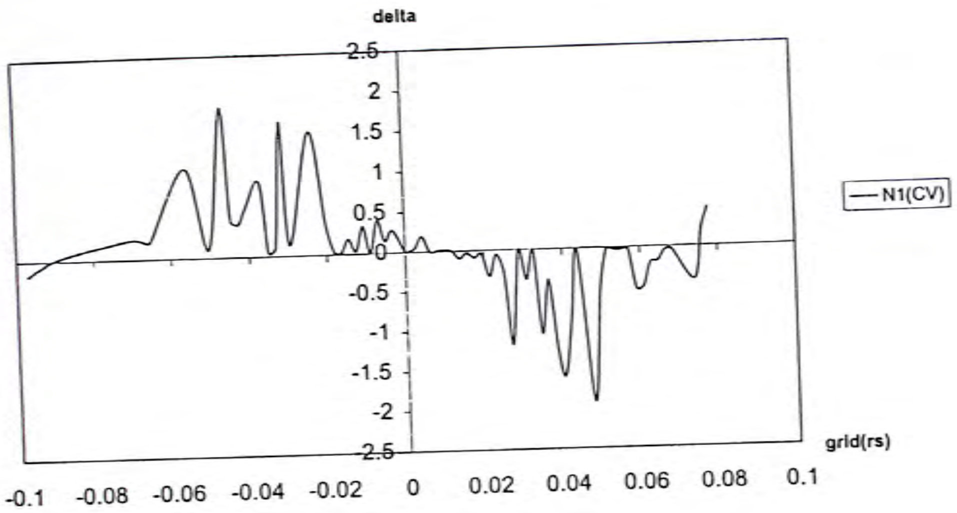

Figure 10 Deltas from Nonparametric Model (N1 with Plug-In Bandwidth)
for Short-term and Out-the-money Series (In-Sample)

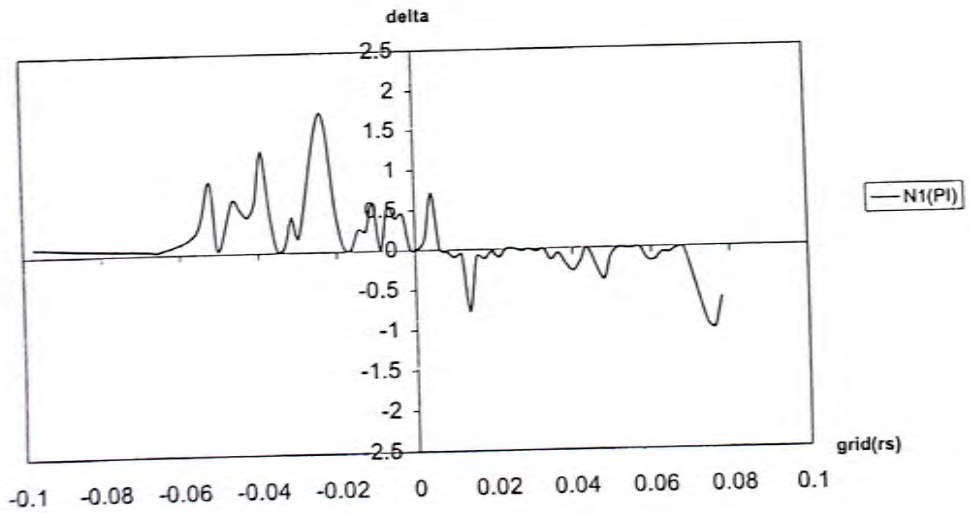

Figure 11 Deltas from Nonparametric Model (N2 with Cross-Validation Bandwidth)
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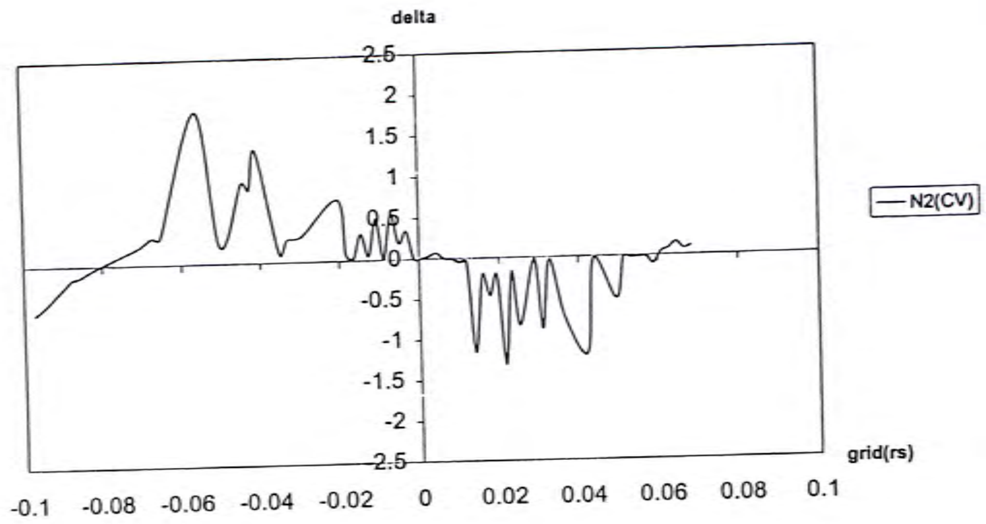

Figure 12 Deltas from Nonparametric Model (N2 with Plug-In Bandwidth)
for Short-term and Out-the-money Series (In-Sample)

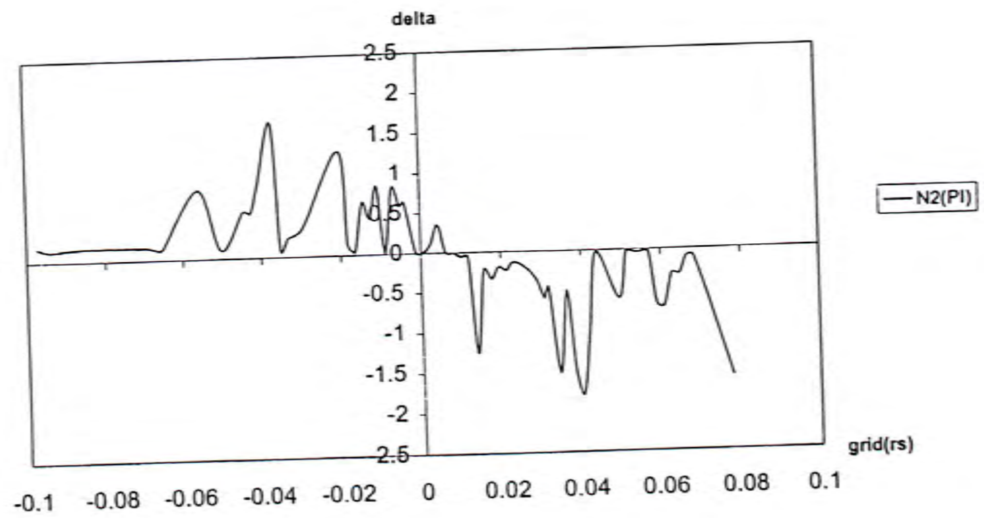

Figure 13 Deltas from Nonparametric Model (N1 with Cross-Validation Bandwidth) for Short-term and At-the-money Series (In-Sample)

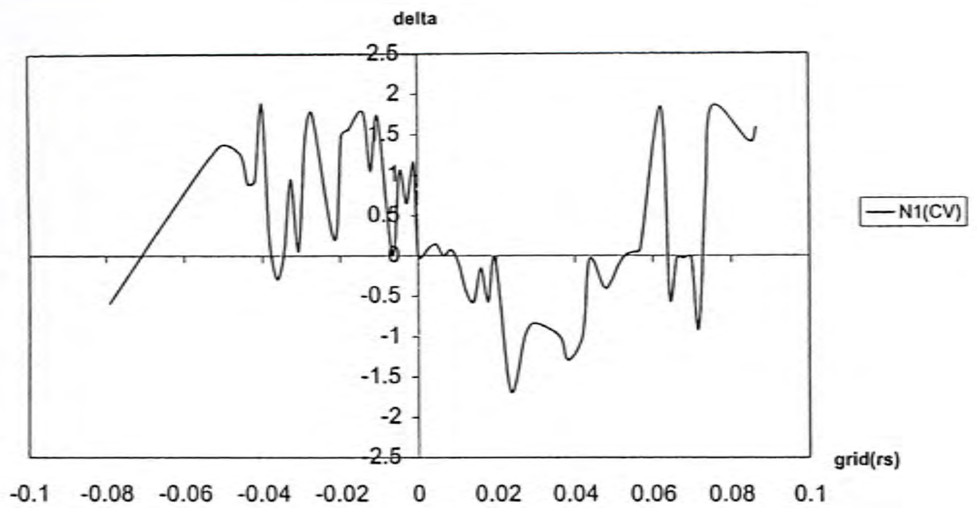

Figure 15 Deltas from Nonparametric Model (N2 with Cross-Validation Bandwidth) for Short-term and At-the-money Series (In-Sample)

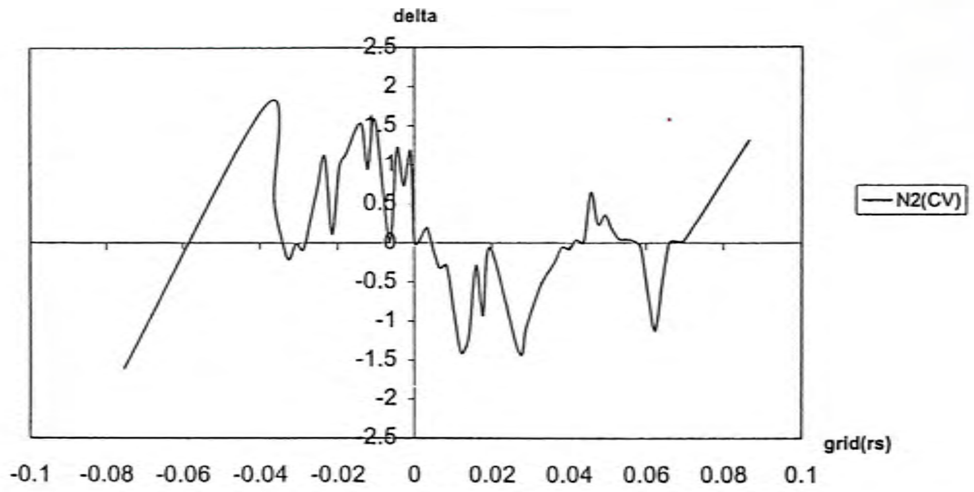

Figure 14 Deltas from Nonparametric Model (N1 with Plug-In Bandwidth) for Short-term and At-the-money Series (In-Sample)

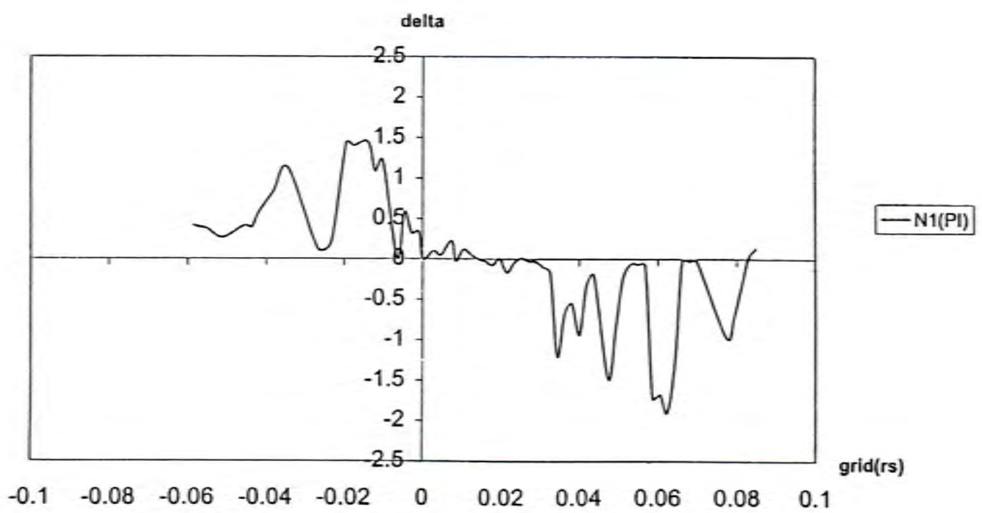

Figure 16 Deltas from Nonparametric Model (N2 with Plug-In Bandwidth) for Short-term and At-the-money Series (In-Sample)

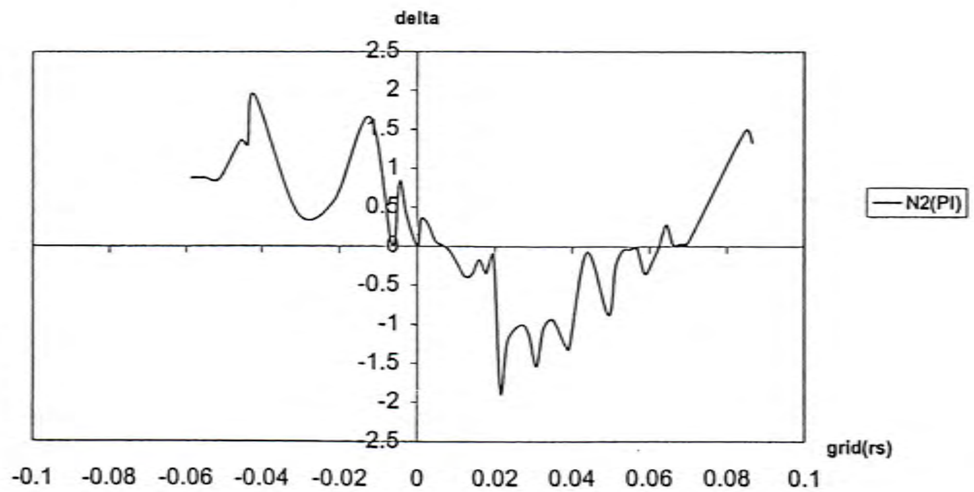

Figure 17 Deltas from Nonparametric Model (N1 with Cross-Validation Bandwidth) for Short-term and In-the-money Series (In-Sample)

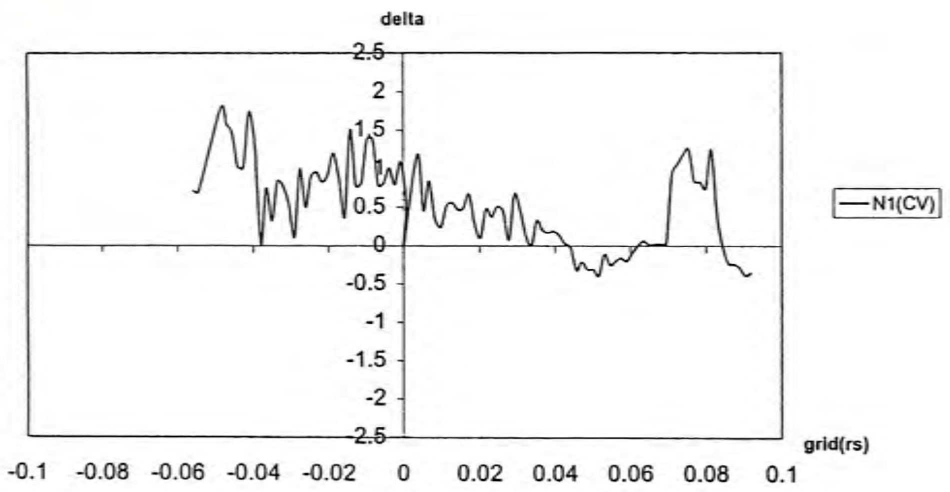

Figure 18 Deltas from Nonparametric Model (N1 with Plug-In Bandwidth) for Short-term and In-the-money Series (In-Sample)

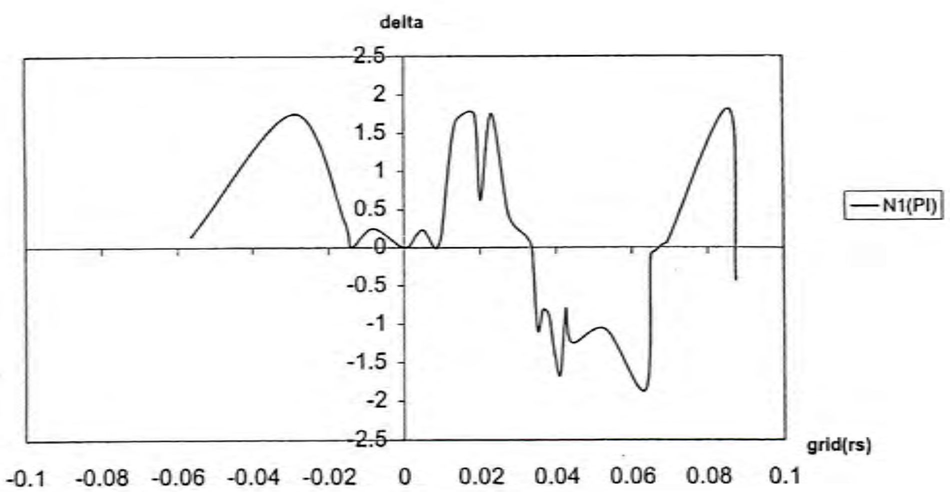

Figure 19 Deltas from Nonparametric Model (N2 with Cross-Validation Bandwidth) for Short-term and In-the-money Series (In-Sample)

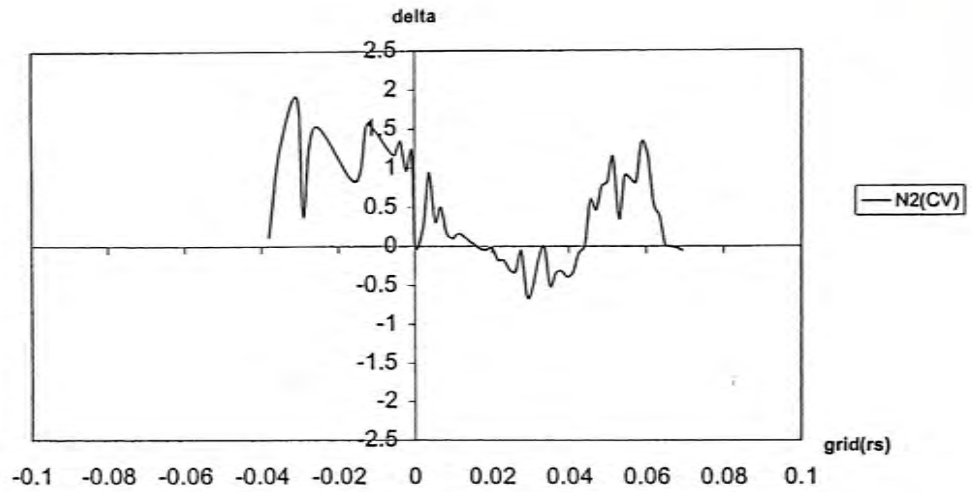

Figure 20 Deltas from Nonparametric Model (N2 with Plug-In Bandwidth) for Short-term and In-the-money Series (In-Sample)

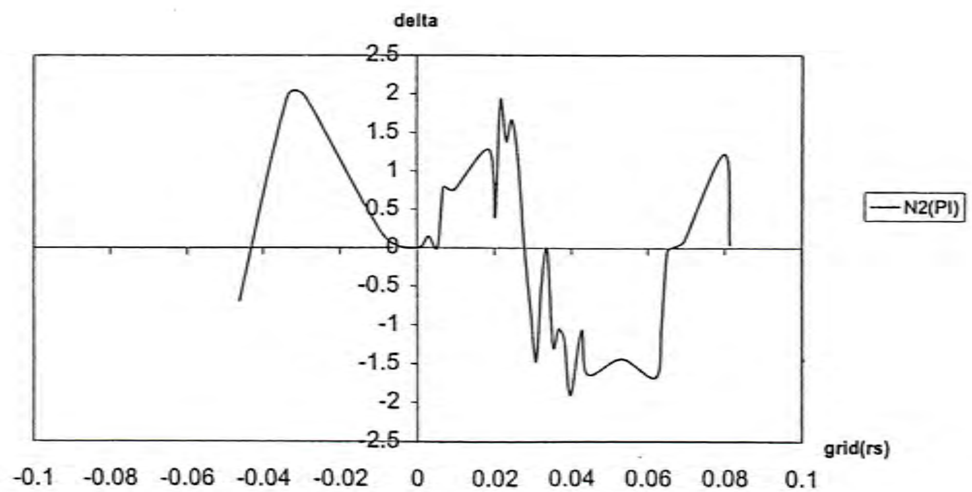

Figure 21 Deltas from Nonparametric Model (N1 with Cross-Validation Bandwidth)
for Long-term and Out-the-money Series (In-Sample)

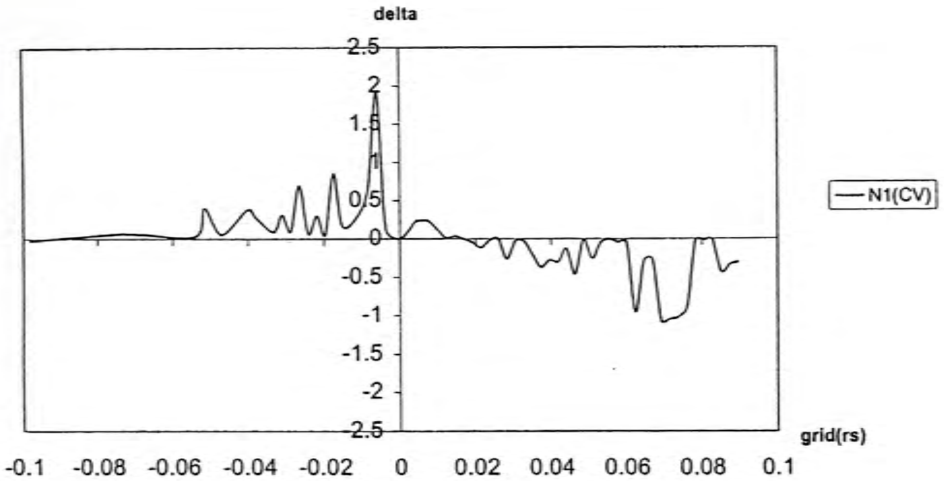

Figure 22 Deltas from Nonparametric Model (N1 with Plug-In Bandwidth)
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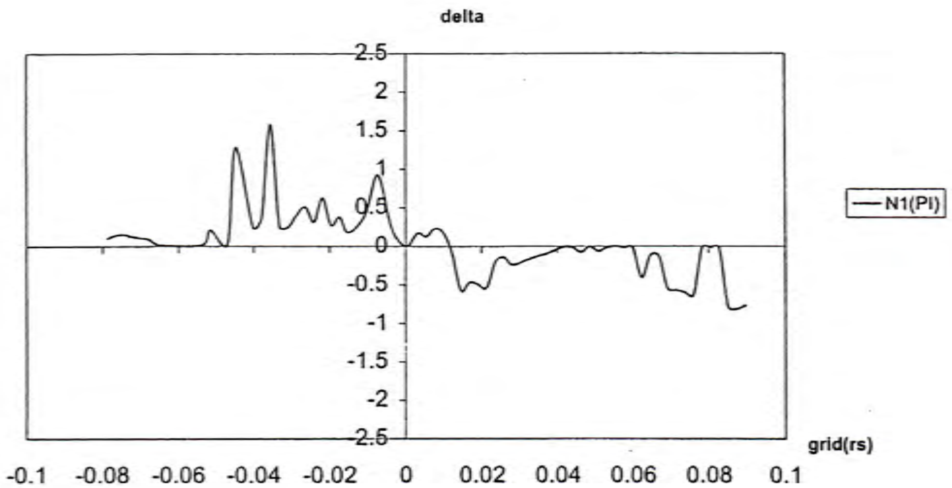

Figure 23 Deltas from Nonparametric Model (N2 with Cross-Validation Bandwidth)
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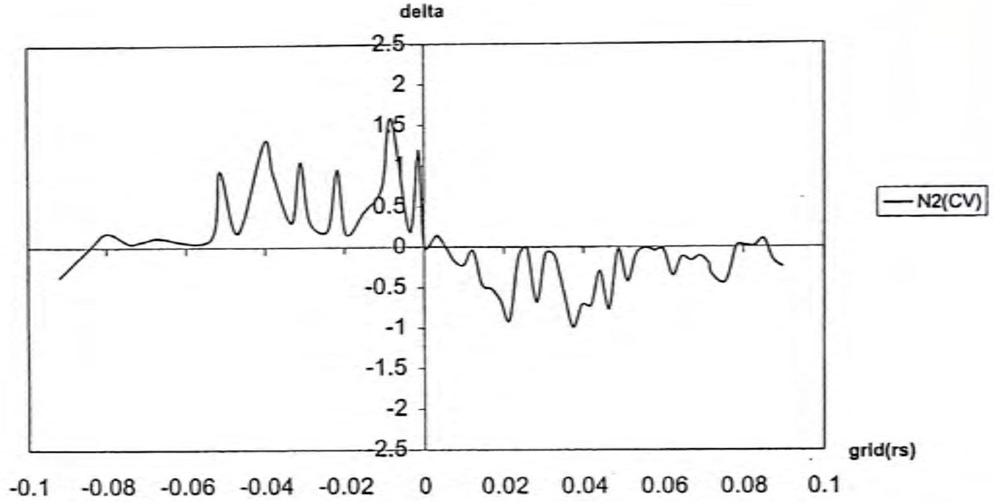

Figure 24 Deltas from Nonparametric Model (N2 with Plug-In Bandwidth)
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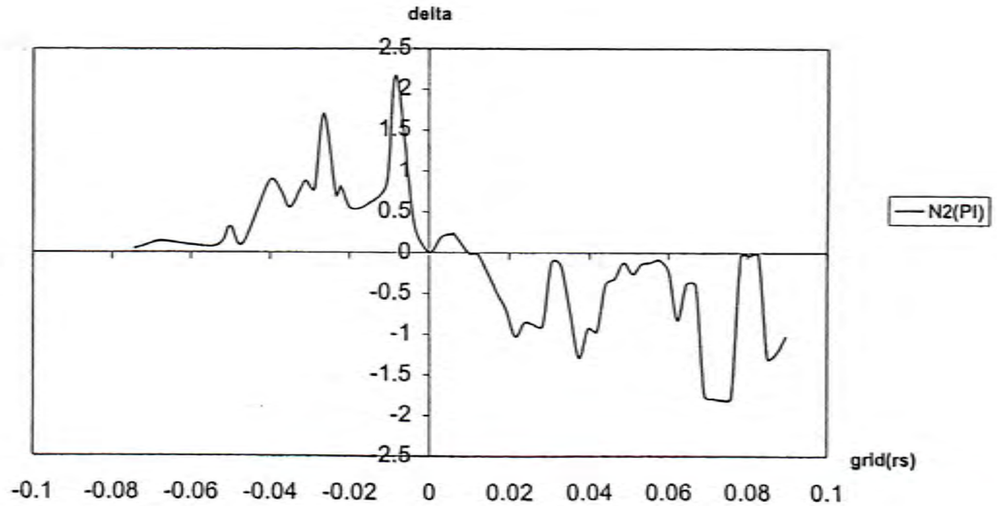


Figure 25 Deltas from Nonparametric Model (N1 with Cross-Validation Bandwidth) for Long-term and At-the-money Series (In-Sample)

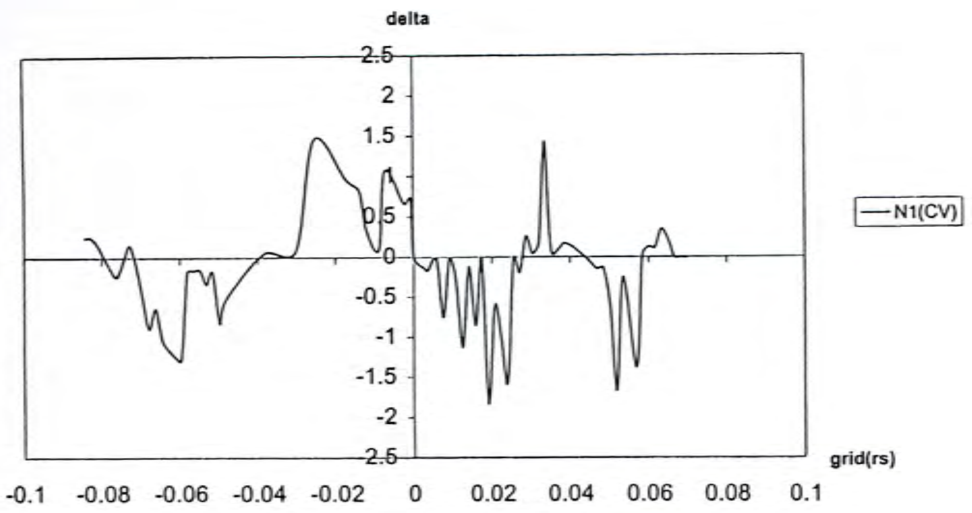


Figure 26 Deltas from Nonparametric Model (N1 with Plug-In Bandwidth) for Long-term and At-the-money Series (In-Sample)

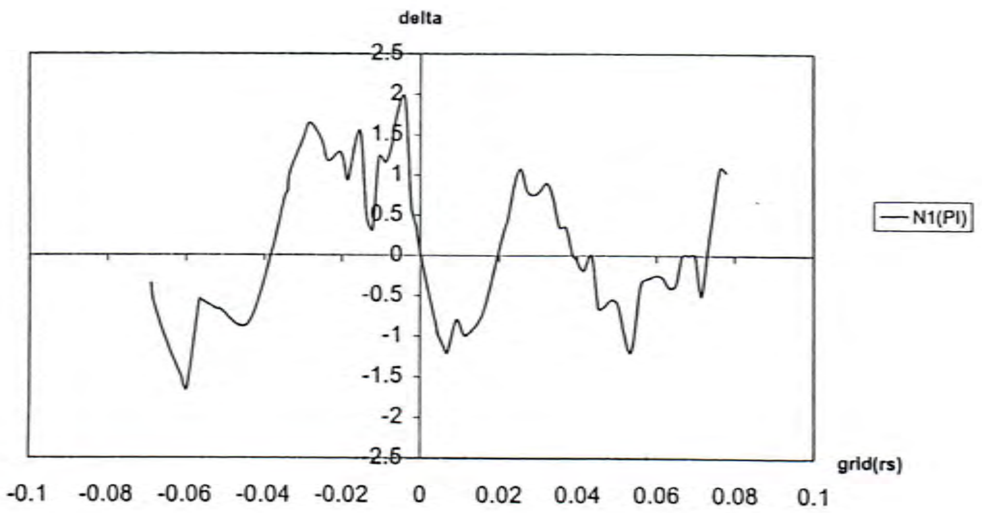


Figure 27 Deltas from Nonparametric Model (N2 with Cross-Validation Bandwidth) for Long-term and At-the-money Series (In-Sample)

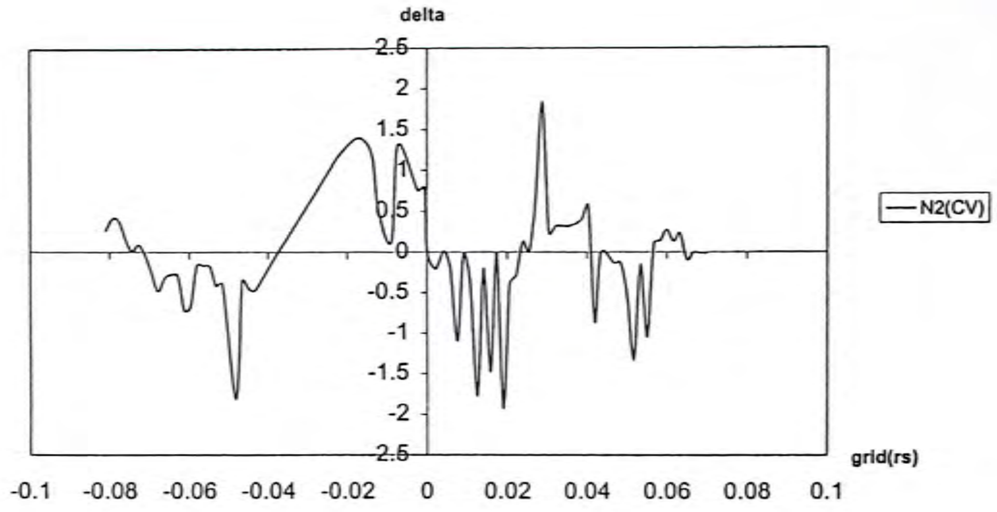


Figure 28 Deltas from Nonparametric Model (N2 with Plug-In Bandwidth) for Long-term and At-the-money Series (In-Sample)

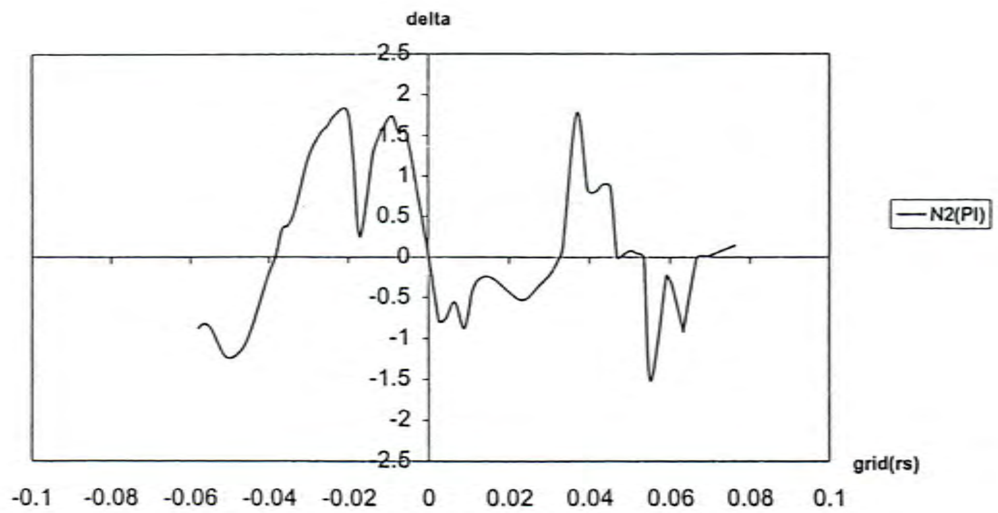


Figure 29 Deltas from Nonparametric Model (N1 with Cross-Validation Bandwidth)
for Long-term and In-the-money Series (In-Sample)

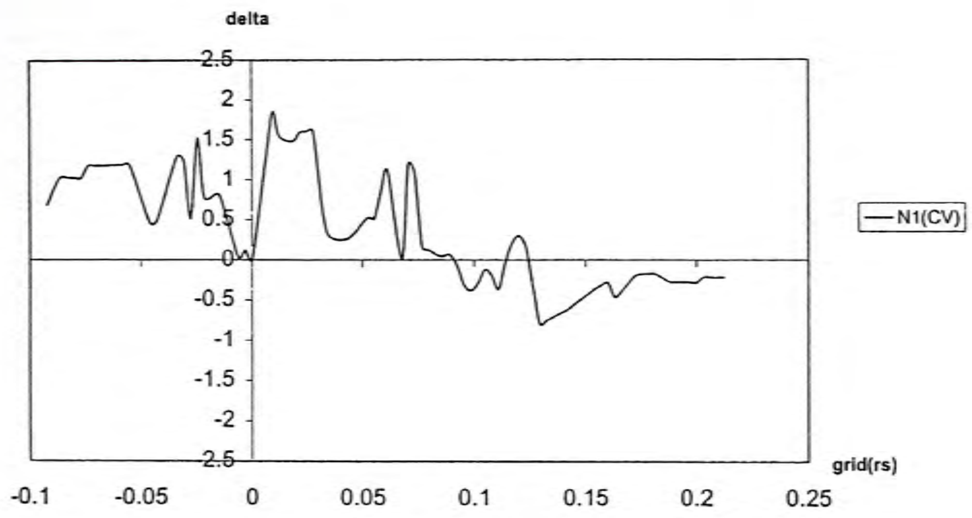


Figure 30 Deltas from Nonparametric Model (N1 with Plug-In Bandwidth)
for Long-term and In-the-money Series (In-Sample)

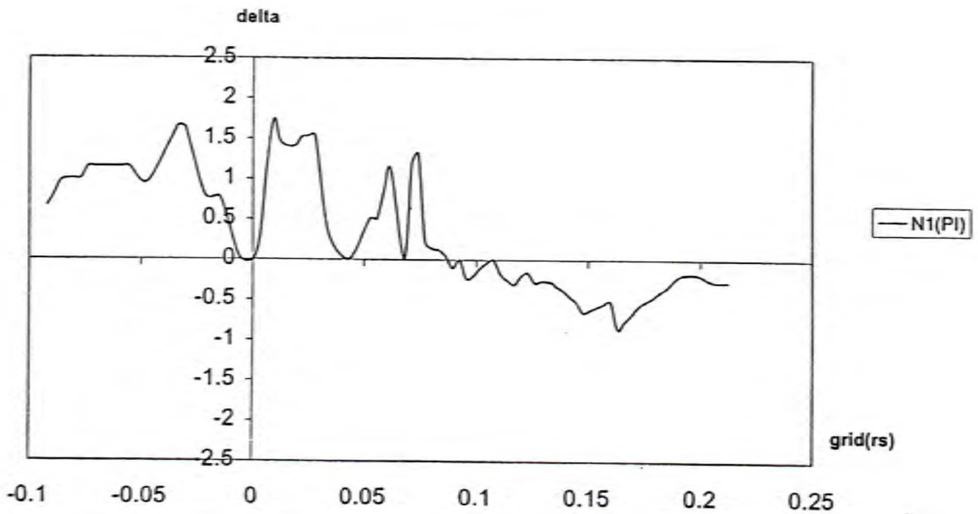


Figure 31 Deltas from Nonparametric Model (N2 with Cross-Validation Bandwidth)
for Long-term and In-the-money Series (In-Sample)

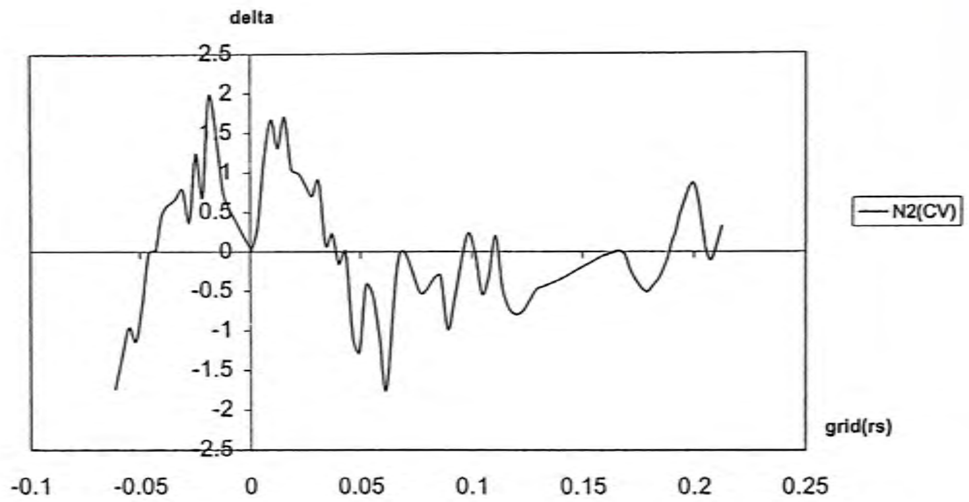


Figure 32 Deltas from Nonparametric Model (N2 with Plug-In Bandwidth)
for Long-term and In-the-money Series (In-Sample)

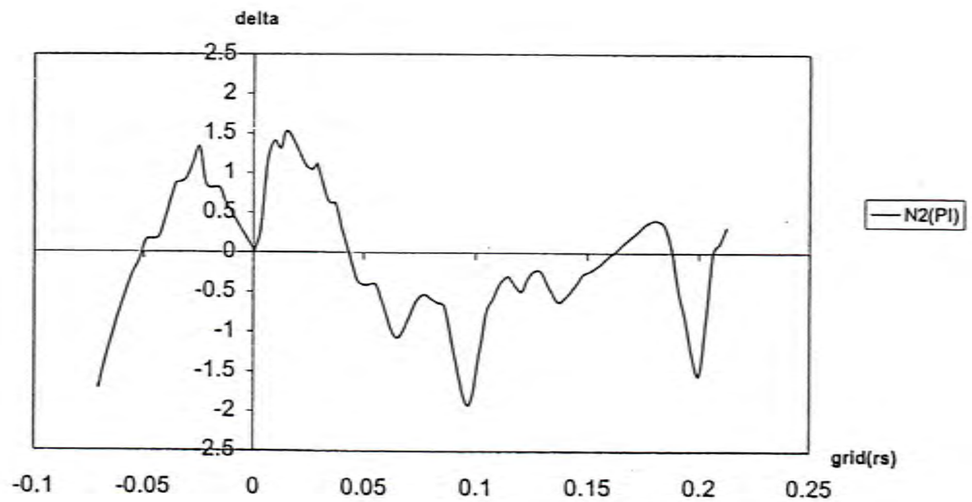


Figure 33 The Variation of Deltas from Local Parametric Models with the Time-to-maturity for Short-term and Out-the-money Options (June 2000)

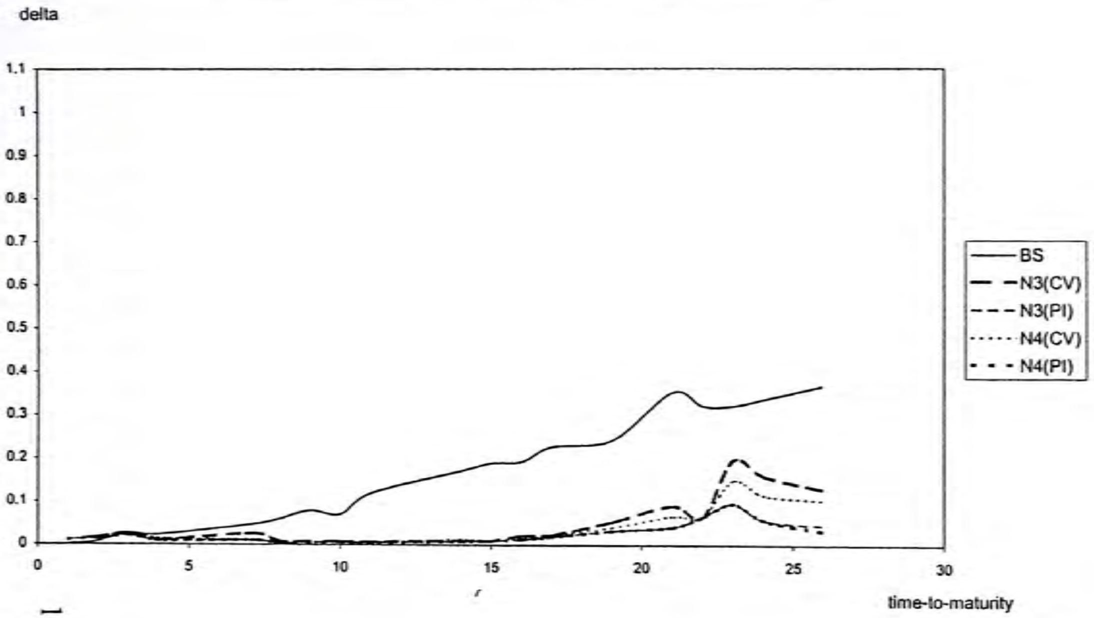


Figure 34 The Variation of Deltas from Local Parametric Models with the Time-to-maturity for Short-term and At-the-money Options (June 2000)

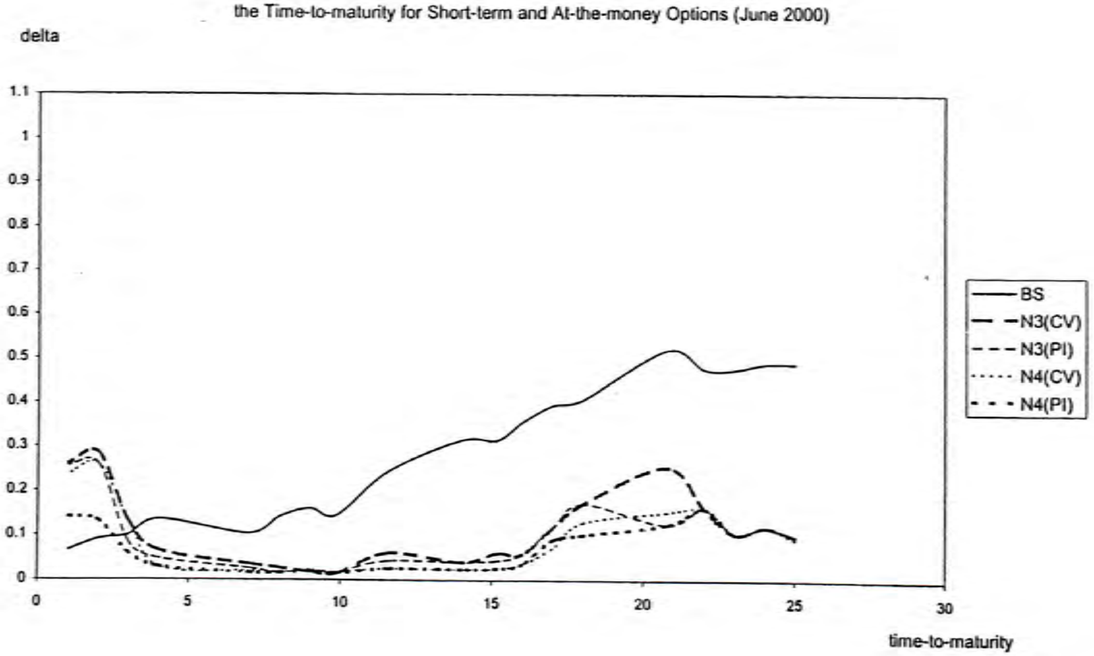


Figure 35 The Variation of Deltas from Local Parametric Models with the Time-to-maturity for Short-term and In-the-money Options (June 2000)

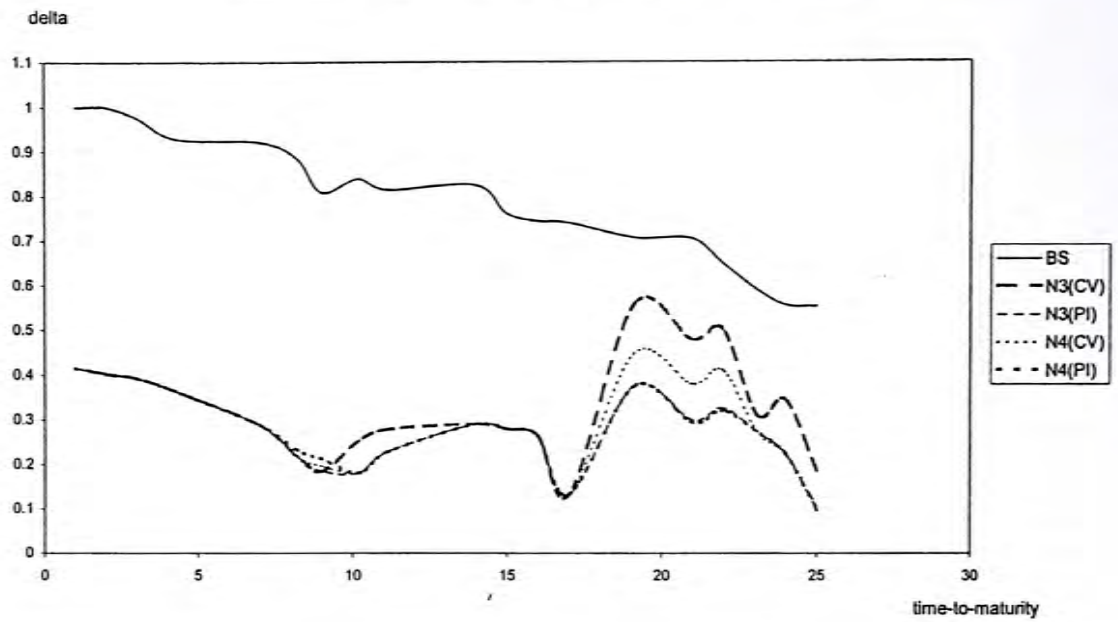


Figure 36 The Variation of Deltas from Local Parametric Models with the Time-to-maturity for Long-term and Out-the-money Options (June 2000)

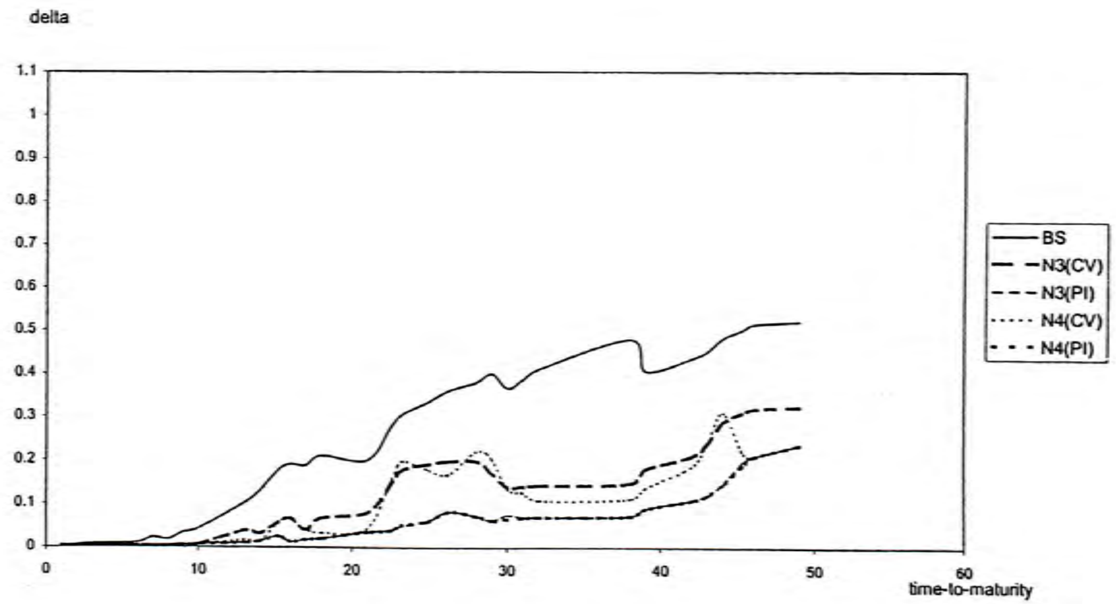


Figure 37 The Variation of Deltas from Local Parametric Models with the Time-to-maturity for Long-term and At-the-money Options (June 2000)

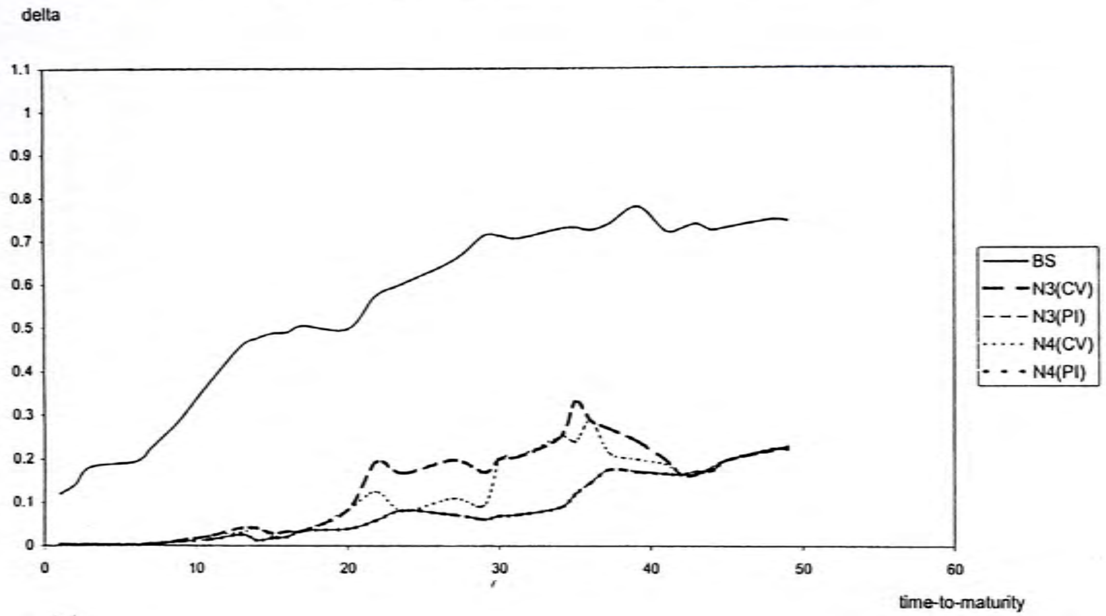
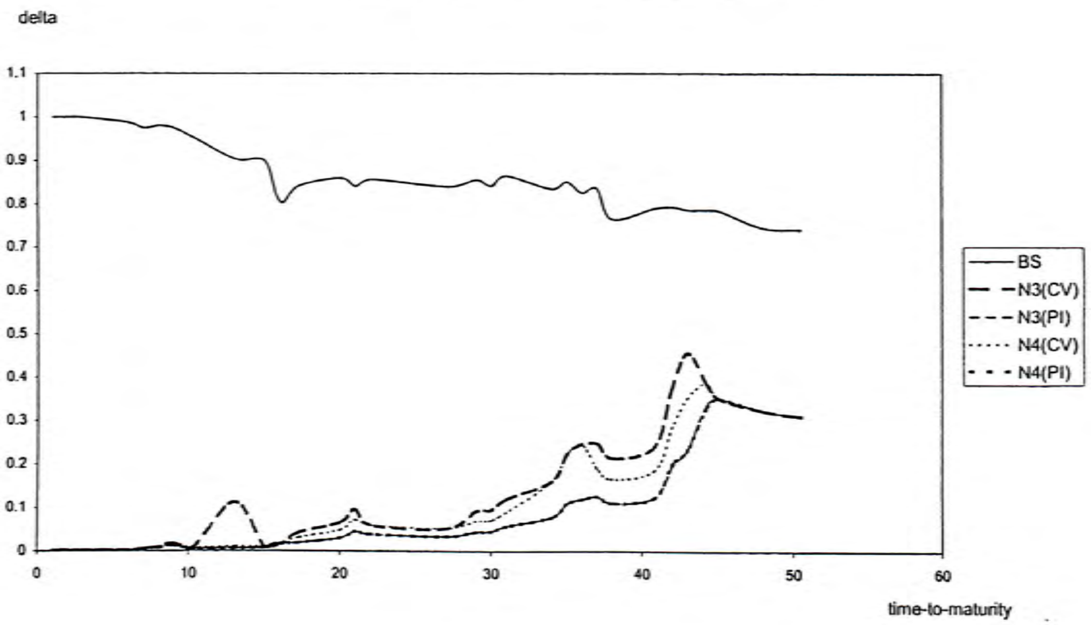


Figure 38 The Variation of Deltas from Local Parametric Models with the Time-to-maturity for Long-term and In-the-money Options (June 2000)



APPENDIX

A Proof of Black and Scholes Formula

Before the proof of the Black and Scholes formula is presented, we first show the proof of the following key result.

Key Result

If V is lognormally distributed and the standard deviation of $\ln V$ is s then

$$E[\max(V-X,0)] = E(V)N(d_1) - XN(d_2); \tag{29}$$

where

$$d_1 = \frac{\ln[E(V)/X] + s^2/2}{s},$$
$$d_2 = \frac{\ln[E(V)/X] - s^2/2}{s},$$

and E denotes expected value.

Proof of Key Result

Define $g(V)$ as the probability density function of V. It follows that

$$E[\max(V-X,0)] = \int_X^\infty (V-X)g(V)dV. \tag{30}$$

The variable $\ln V$ is normally distributed with standard deviation s. From the properties of the lognormal distribution, the mean of $\ln V$ is m where

$$m = \ln[E(V)] - s^2/2. \tag{31}$$

The standardized $\ln V$ is

$$Q = \frac{\ln V - m}{s}. \tag{32}$$

This variable is normally distributed with a mean of zero and a standard deviation of one. The density function for Q, say $h(Q)$, is

$$h(Q) = \frac{1}{\sqrt{2\pi}} e^{-Q^2/2}.$$

Using Equation (32) to convert the expression on the right-hand side of Equation (30) from an integral over V to an integral over Q. The expected value now becomes

$$\hat{E}[\max(V - X, 0)] = \int_{(\ln X - m)/s}^{\infty} \left(e^{Qs+m} - X\right) h(Q) dQ$$

or

$$\hat{E}[\max(V - X, 0)] = \int_{(\ln X - m)/s}^{\infty} e^{Qs+m} h(Q) dQ - X \int_{(\ln X - m)/s}^{\infty} h(Q) dQ. \tag{33}$$

Now

$$e^{Q_s + m} h(Q) = \frac{1}{\sqrt{2\pi}} e^{(-Q^2 + 2Qs + 2m)/2};$$

$$= \frac{1}{\sqrt{2\pi}} e^{[-(Q-s)^2 + 2m + s^2]/2};$$

$$= \frac{e^{m + s^2/2}}{\sqrt{2\pi}} e^{[-(Q-s)^2]/2};$$

$$= e^{m + s^2/2} h(Q - s).$$

This means that Equation (33) becomes

$$\hat{E}[\max(V - X, 0)] = e^{m + s^2/2} \int_{(\ln X - m)/s}^{\infty} h(Q - s) dQ - X \int_{(\ln X - m)/s}^{\infty} h(Q) dQ. \tag{34}$$

If we define $N(x)$ as the probability distribution function of a variable with a mean of zero and a standard deviation of one, the first integral in Equation (34) is

$$1-N[(\ln X-m)/s-s]$$

or

$$N[(-\ln X-m)/s-s].$$

Substituting for m from Equation (31) to the above equation. It becomes

$$N\left[\frac{\ln[E(V)/X]+s^2/2}{s}\right]=N(d_1).$$

Similarly, the second integral in Equation (34) is $N(d_2)$. Equation (34), therefore, becomes

$$\hat{E}[\max(V-X,0)]=e^{m+s^2/2}N(d_1)-XN(d_2).$$

Substituting m from Equation (31) to the above equation, the key result follows.

The Black and Scholes Result

We now consider a call option on a non-dividend-paying stock maturing at time T. The strike price is X, the risk-free rate is r, the current stock price is S_0, and the volatility is σ. The expected value of the call option at maturity in a risk-neutral world is

$$\hat{E}[\max(S_T-X,0)];$$

where S_T is the stock price at time T and $\hat{E}$ denotes expectations in a risk-neutral world.

From the risk-neural valuation argument, this expectation discounted at the risk-free rate of interest gives the European call option price, c, namely,

$$c = e^{-rT} \hat{E}[\max(S_T - X, 0)]. \tag{35}$$

Under the stochastic process assumed by Black and Scholes, S_T is lognormal and follows geometric Brownian motion. The expected value of S_T, $\hat{E}(S_T)$, is given by $\hat{E}(S_T) = S_0 e^{rT}$ and the standard deviation of $\ln S_T$ is $\sigma\sqrt{T}$.

From the key result proved above, Equation (35) implies

$$c = e^{-rT}\left[S_0 e^{rT} N(d_1) - XN(d_2)\right]$$

or

$$c = S_0 N(d_1) - Xe^{-rT} N(d_2);$$

where

$$d_1 = \frac{\ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}};$$
$$d_2 = \frac{\ln(S_0/X) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

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